Chapter 5

Mixers

A mixer is a device that multiplies two signals. In RF and microwave design, mixers are used to upconvert or downconvert a signal in frequency. If a narrowband signal centered around a carrier frequency is mixed with a sinusoidal signal at the same frequency as the carrier, one of the mixing products that appears at the output is the narrowband signal centered at DC. Often, instead of mixing a signal all the way to DC, it is common to mix to an intermediate frequency (IF) such as 70 MHz or 140 MHz that is much smaller than the carrier frequency. Further processing and detection of the information carried in the signal can then be done at the lower frequency with much simpler circuits than would be required at the original carrier frequency.

RF mixers can also be used for modulation/demodulation, although this is uncommon since most modern communication signals use complex modulations which would require two mixers with nearly identical phase performance. Since this is difficult to construct at high frequencies, modulation is usually done at a lower IF frequency and then the signal is upconverted to a center frequency in the microwave band.

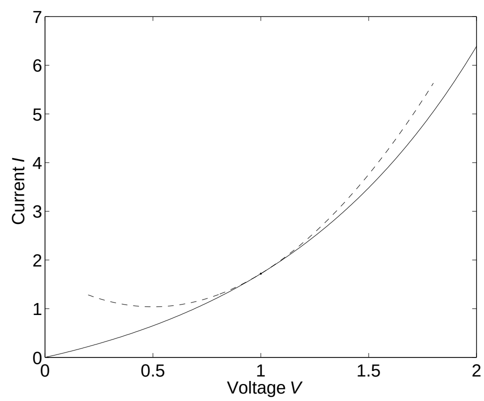

Figure 5.1: Nonlinear diode current/voltage characteristic. Solid line: Eq. (5.1) with $\alpha = I_s = 1$. Dashed line: second order approximation using the first three terms of Eq. (5.2).

One way to perform mixing is take two signals of similar strength, add them together, and then run them through a diode. To see how this works mathematically, consider that a diode has a voltage-current relationship of

$$I(V) = I_s(e^{\alpha V} - 1) \tag{5.1}$$

If the voltage is written as $V = V_o + v$, where V_o is a DC bias voltage and v is a small AC term, then we can write a Taylor series of the current as

$$I(V_o + v) = I(V_o) + v \left. \frac{dI}{dV} \right|_{V=V_o} + \frac{v^2}{2} \left. \frac{d^2 I}{dV^2} \right|_{V=V_o} + \cdots \tag{5.2}$$

where we can neglect higher order terms as long as v is small. Defining $dI/dV = 1/R_j$ at $V = V_o$, where R_j is the small-signal junction resistance, then $d^2I/dV^2 = \alpha/R_j$ at $V = V_o$. If $v = v_1 + v_2$, then the second-order term in the Taylor series produces

$$\frac{v^2}{2} \left. \frac{d^2 I}{dV^2} \right|_{V=V_o} = \frac{\alpha}{2R_j}(v_1^2 + 2v_1 v_2 + v_2^2) \tag{5.3}$$

The middle term performs the multiplication we desire. So, all we have to do is match the diode to the feedline in order to maximize the voltage across the diode junction and thereby maximize the signal strength of the multiplied signal.

This nonlinear diode relationship also indicates how to do power detection. We often need to do power sampling in a wireless communication system, for example, when the gain of the receiver must be be automatically varied depending on the received signal strength. If the input voltage is $v(t) = A\cos\omega t$, then the squaring operation leads to

$$\frac{v^2}{2} \left. \frac{d^2 I}{dV^2} \right|_{V=V_o} = \frac{\alpha A^2}{4R_j} \left[1 + \cos(2\omega t)\right] \tag{5.4}$$

Filtering out the term at $2\omega t$ leaves a DC term that is proportional to the received signal power (voltage squared).

5.1 Switching (Sampling) Mixers

The diode mixer described above is relatively straightforward to design and build, and is used in some cases for special purpose designs, but most commercial mixers use an different approach based on switching. Switching mixers are not as intuitive as nonlinear mixing, but can be readily analyzed using Fourier analysis.

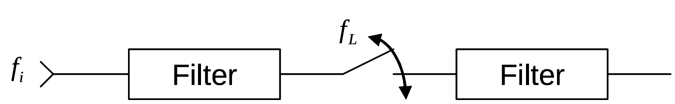

Figure 5.2: A simple sampling mixer configuration.

Consider a system with a high-speed switch that samples an input signal at frequency f_i at a sample rate of f_L. The system is shown in Fig. 5.2. Assume that the switch is an ideal sampling device for now, which means that it is closed only instantaneously. The sampling function is therefore a sequence of delta functions. The spectrum of the sampling function will also be a sequence of delta functions, as shown in Fig. 5.3.

Since we are multiplying the signal by $v_L(t)$, we are convolving in the frequency domain. This will replicate the signal spectrum at f_L intervals in the frequency domain. We can then filter out images of the spectrum that we do not want. Note that we call v_L the *Local Oscillator (LO)* signal. The high frequency signal is typically called the *Radio Frequency (RF)* signal, and the low frequency signal the *Intermediate Frequency*

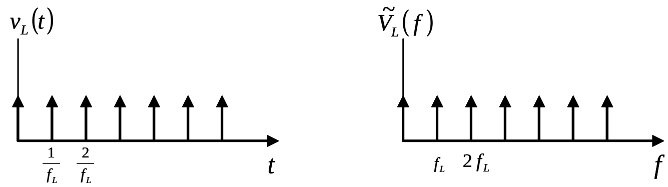

Figure 5.3: Sampling voltage and spectrum.

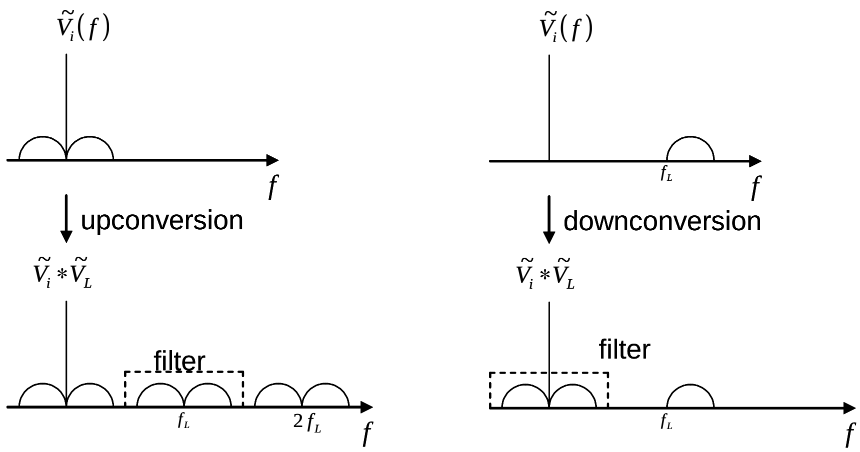

Figure 5.4: Sampling mixer up- and downconversion of a signal.

(IF) signal. If the RF is the input, IF is the output (downconversion). If the IF is the input, the RF is the output (upconversion). Upconversion and downconversion are represented in the spectral domain in Fig. 5.4.

Now, suppose $v_L(t)$ is not a train of impulses, but rather some periodic function with a period of $T_L = 1/f_L$. $\tilde{V}_L(f)$ will still be impulses at nf_L. However, they will not be equal amplitude, since the spectrum of the sampling function decays with frequency. This is what we obtain using a diode as the switching device (Fig. 5.5).

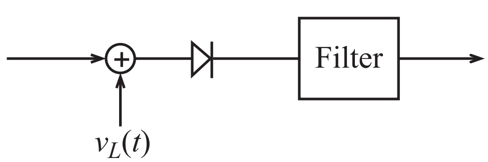

Figure 5.5: Diode switching mixer.

The local oscillator is a strong signal, which when positive causes the diode to conduct (switch closed), when negative causes the diode to be reverse biased (switch open). If the required turn-on voltage of the diode is v_d, then the diode is on when the switching signal is greater than v_d, as shown in Fig. 5.6.

For simplicity, we will assume that the switching voltage is strong enough that we can neglect the tiny turn-on voltage V_d, which means that when $v_L > 0$ the diode conducts. Mathematically, we can write

$$v_o(t) = [v_L(t) + v_i(t)]\, S_s(t) \tag{5.5}$$

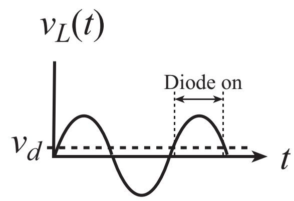

Figure 5.6: Diode voltage over one cycle of the switching signal.

where

$$\begin{aligned} S_s(t) &= \begin{cases} 1 & v_L > 0 \\ 0 & v_L < 0 \end{cases} \\ &= \frac{1}{2} + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin\left(\frac{n\pi}{2}\right)\cos(n\omega_L t) \end{aligned} \tag{5.6}$$

Therefore, if we assume that $v_i(t) = V_i \cos\omega_i t$ and $v_L(t) = V_L \cos\omega_L t$, then

$$\begin{aligned} v_o(t) &= \frac{1}{2}v_L(t) + \frac{1}{2}v_i(t) + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin\left(\frac{n\pi}{2}\right)\{V_L\cos(n\omega_L t)\cos(\omega_L t) + V_i\cos(n\omega_L t)\cos(\omega_i t)\} \\ &= \frac{1}{2}v_L(t) + \frac{1}{2}v_i(t) + \frac{1}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin\left(\frac{n\pi}{2}\right)\{V_L\cos[(n-1)\omega_L t] + V_L\cos[(n+1)\omega_L t] + \\ &\qquad V_i\cos[(n\omega_L - \omega_i)t] + V_i\cos[(n\omega_L + \omega_i)t]\} \end{aligned} \tag{5.7}$$

Note that the $\sin(n\pi/2)$ term is zero for n even. Therefore, we will have signals at

1. ω_i
2. ω_L
3. $m\omega_L$ for m even
4. $n\omega_L \pm \omega_i$ for n odd

The desired term will be either $\omega_L + \omega_i$ for upconversion or $\omega_L - \omega_i$ for downconversion. The remaining undesired components will need to be filtered out.

5.2 Single Balanced Mixers

The simple mixer introduced above is a single-ended mixer. We have shown that this mixer produces a large variety of undesired signals. If we use a more balanced configuration, then some of these undesired signals can be suppressed. For example, consider the single-balanced mixer shown below:

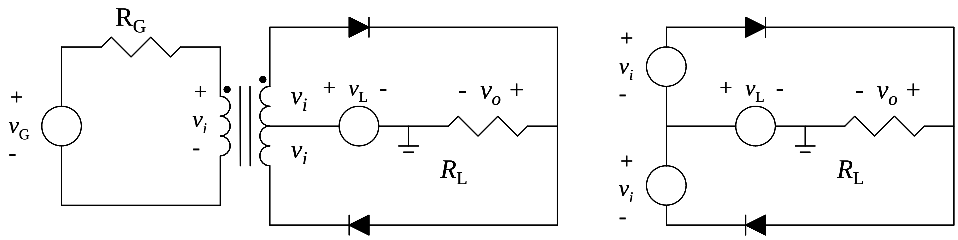

Figure 5.7: Single-balanced diode switching-type mixer.

For this circuit,

$$\begin{aligned} v_o &= \begin{cases} v_L + v_i & v_L > 0 \\ v_L - v_i & v_L < 0 \end{cases} \\ &= v_L + v_i S_b(t) \end{aligned} \tag{5.8}$$

where

$$\begin{aligned} S_b(t) &= \begin{cases} +1 & v_L > 0 \\ -1 & v_L < 0 \end{cases} \\ &= \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin\left(\frac{n\pi}{2}\right)\cos(n\omega_L t) \end{aligned} \tag{5.9}$$

So,

$$v_o(t) = v_L(t) + \frac{2V_i}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin\left(\frac{n\pi}{2}\right)\{\cos[(n\omega_L - \omega_i)t] + \cos[(n\omega_L + \omega_i)t]\} \tag{5.10}$$

We now have signals at

1. ω_L
2. $n\omega_L \pm \omega_i$ for n odd

The balanced configuration has removed many of our undesired components. This tends to be a good choice for downconversion, since $\omega_L >> \omega_L - \omega_i$, so it is easy to filter out the undesirable signals at ω_L and $n\omega_L \pm \omega_i$ for $n > 1$.

If we augment this single-balanced design as follows: we can ALSO suppress the LO. In this case,

$$v_o(t) = v_i(t)S_s(t) = \frac{1}{2}v_i(t) + \frac{V_i}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin\left(\frac{n\pi}{2}\right)\{\cos[(n\omega_L - \omega_i)t] + \cos[(n\omega_L + \omega_i)t]\} \tag{5.11}$$

This is a good choice for upconversion since $\omega_L >> \omega_i$.

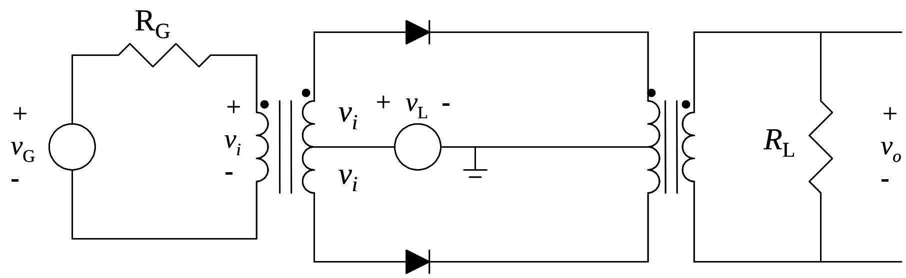

Figure 5.8: Alternate single-balanced diode switching-type mixer.

5.3 Double Balanced Mixer

To suppress both the signal and LO frequencies, we must go to a double balanced design:

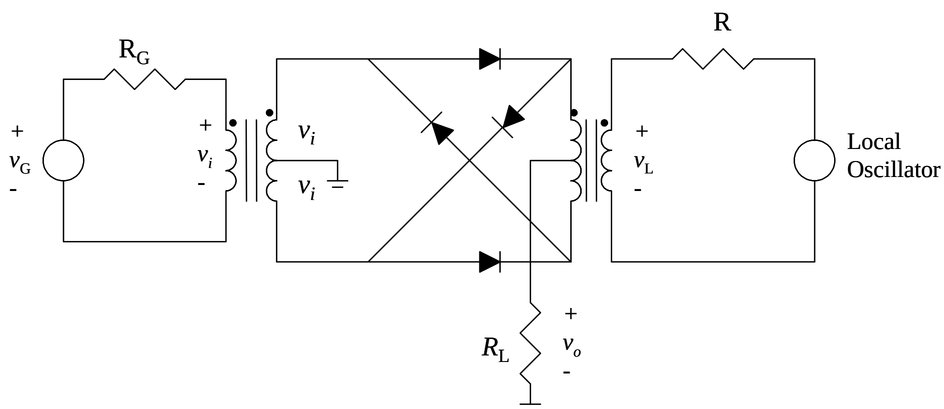

Figure 5.9: Double-balanced diode switching-type mixer.

For $v_L > 0$, we can simplify the circuit by removing the diodes that are off:

For the simplified circuit ($v_L > 0$):

$$v_i - (i_1 - i_2)R_L + v_L - r_d i_1 = 0 \tag{5.12}$$

$$v_i - (i_1 - i_2)R_L - v_L + r_d i_2 = 0 \tag{5.13}$$

Adding these equations leads to

$$2v_i - 2R_L(i_1 - i_2) - r_d(i_1 - i_2) = 0 \tag{5.14}$$

$$i_1 - i_2 = \frac{v_i}{R_L + r_d/2} = -\frac{v_o}{R_L} \tag{5.15}$$

For $v_L > 0$, we have

$$\frac{v_o}{v_i} = -\frac{R_L}{R_L + r_d/2} \tag{5.16}$$

A similar analysis for $v_L < 0$ leads to

$$\frac{v_o}{v_i} = \frac{R_L}{R_L + r_d/2} \tag{5.17}$$

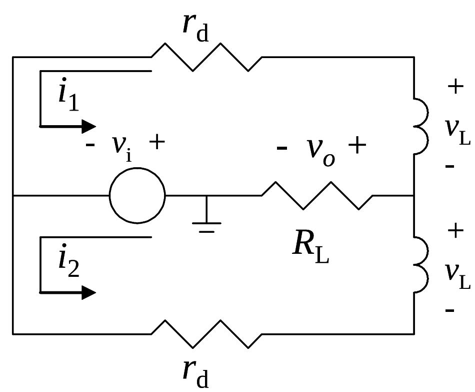

Figure 5.10: Simplified double-balanced mixer circuit for $v_L > 0$.

Combining these two results leads to

$$\begin{aligned} v_o &= \frac{R_L}{R_L + r_d/2} v_i S_b(t) \\ &= \frac{R_L}{R_L + r_d/2} \frac{2V_i}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \sin\left(\frac{n\pi}{2}\right) \{\cos[(n\omega_L - \omega_i)t] + \cos[(n\omega_L + \omega_i)t]\} \end{aligned} \quad (5.18)$$

This shows that the double balanced mixer eliminates both the LO and input signals at the output.

5.3.1 Conversion Loss

For the double-balanced mixer,

$$R_{in} = \frac{v_i}{i_1 - i_2} = R_L + r_d/2 \approx R_L \quad (5.19)$$

The maximum available power from the source is

$$P_i = \frac{V_p^2}{8R_L} \quad (5.20)$$

where V_p is the peak value of the source sinusoidal signal. The peak of the output voltage in a single sideband is then

$$V_o = \frac{2V_i}{\pi} = \frac{V_p}{\pi} \quad (5.21)$$

due to the voltage division of V_p. The output power is

$$P_o = \frac{V_p^2}{2\pi^2 R_L} \quad (5.22)$$

and the conversion loss is

$$L = \frac{P_i}{P_o} = \frac{\pi^2}{4} \quad (5.23)$$

$$L_{\text{dB}} = 10 \log\left(\frac{\pi^2}{4}\right) = 3.92 \text{ dB} \approx 4 \text{ dB} \quad (5.24)$$

For the single-balanced mixer,

$$L_{\text{dB}} = 9.94 \text{ dB} \approx 10 \text{ dB} \quad (5.25)$$