Chapter 4

Oscillators

There are several types of sources of microwave signals:

- *Black-body radiation.* All materials give off a small amount of microwave radiation due to black-body or thermal radiation. This effect is used in passive remote sensing and receiver calibration.
- *Microwave tubes*. These sources are typically used for very high power applications.
- *Diodes.* A diode source converts DC into microwave energy by making use of a negative resistance voltage-current characteristic (one-port oscillators).
- *Transistors.* Transistor microwave oscillators are similar in principle to low frequency oscillators—amplifiers with feedback (two-port oscillators).

4.1 Oscillator Basics

In studying the stability of an amplifier design, we recognized that we can have a transistor with an input or output reflection coefficients have a magnitude greater than unity. The question is: how do we use this fact to form an oscillator?

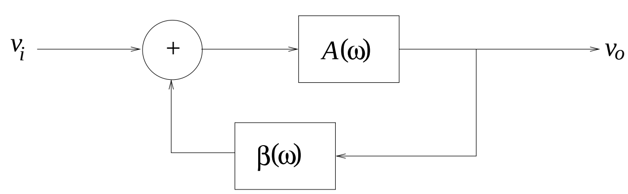

Figure 4.1: Block diagram of a simple feedback network.

By analyzing a simple feedback network, we can find the conditions necessary for oscillation. From the block diagram in Fig. 4.1, the output signal is

$$v_o = A(\omega)[v_i + \beta(\omega)v_o] \tag{4.1}$$

Rearranding this expression gives the transfer function of the system:

$$\frac{v_o}{v_i} = \frac{A(\omega)}{1 - \beta(\omega)A(\omega)} \tag{4.2}$$

For oscillation to occur, we want an output v_o with no input v_i ($v_o/v_i \rightarrow \infty$). This leads to the condition

$$\beta(\omega)A(\omega) = 1 \tag{4.3}$$

We see that the loop gain must have: 1) unity magnitude and 2) $2\pi n$ phase, where n is an integer.

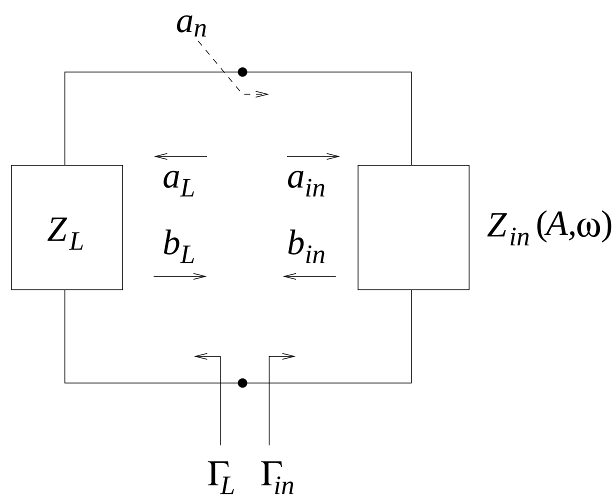

Figure 4.2: Oscillator network with load on the left and active (nonlinear) device on the right. Because the device is nonlinear, its input impedance Z_{in} is a function of the current amplitude A and frequency ω.

Now, let us look at the same concept from a microwave network point of view, as shown in Fig. 4.2. We assume that some noise signal a_n due to thermal or another type of noise is input as an additional forward wave into the active device. The total forward wave into the device is

$$\begin{aligned} a_{\text{in}} &= a_n + \Gamma_{\text{in}}\Gamma_L a_{\text{in}} \\ &= \frac{a_n}{1 - \Gamma_{\text{in}}\Gamma_L} \end{aligned} \tag{4.4}$$

The forward wave into the load is

$$a_L = \Gamma_{\text{in}} a_{\text{in}} = \frac{a_n \Gamma_{\text{in}}}{1 - \Gamma_{\text{in}}\Gamma_L} \tag{4.5}$$

For oscillation, we must have

$$\Gamma_{\text{in}}\Gamma_L = 1 \tag{4.6}$$

We now want to change this to a condition on the load and device impedances:

$$\begin{aligned} \Gamma_{\text{in}}(A,\omega)\Gamma_L(\omega) &= 1 \\ \left[\frac{Z_{\text{in}}(A,\omega) - Z_o}{Z_{\text{in}}(A,\omega) + Z_o}\right]\left[\frac{Z_L(\omega) - Z_o}{Z_L(\omega) + Z_o}\right] &= 1 \\ Z_{\text{in}}Z_L - Z_o(Z_{\text{in}} + Z_L) + Z_o^2 &= Z_{\text{in}}Z_L + Z_o(Z_{\text{in}} + Z_L) + Z_o^2 \\ Z_{\text{in}} + Z_L &= 0 \end{aligned} \tag{4.7}$$

where A is the signal amplitude. Breaking up the impedances into real and reactive parts leads to the conditions

$$R_{\rm in}(A,\omega) + R_L(\omega) = 0 \tag{4.8}$$
$$X_{\rm in}(A,\omega) + X_L(\omega) = 0 \tag{4.9}$$

For the reactive part, we choose $X_L(\omega) = -X_{\rm in}(A,\omega)$. Now, the question is how to pick $R_L(\omega)$ so that oscillation starts up properly and to provide high output power.

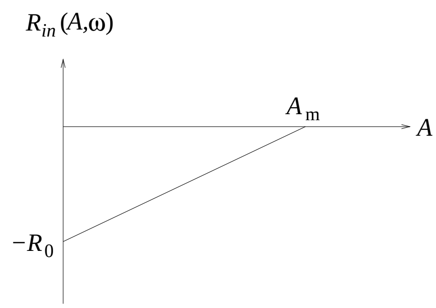

Figure 4.3: Device resistance as a function of current amplitude A.

Let us assume that $R_{\rm in}(A,\omega)$ is linear in A. (Does this mean that the device is linear?) Let us denote the $A = 0$ intercept as $R_{\rm in}(0,\omega) = -R_o$, as shown in Fig. 4.3. The input resistance can be expressed as

$$R_{\rm in}(A,\omega) = -R_o\,[1 - A/A_m] \tag{4.10}$$

where A_m is the output signal level at which $R_{in}(A_m,\omega) = 0$. The power available to the load is

$$P_{\rm avn} = \frac{1}{2}{\rm Re}\{VI^*\} \tag{4.11}$$
$$= \frac{1}{2}|I|^2{\rm Re}\{Z_{\rm in}(A,\omega)\} \tag{4.12}$$
$$= \frac{1}{2}|I|^2 R_{\rm in}(A,\omega) \tag{4.13}$$
$$= -\frac{1}{2}A^2 R_o\,[1 - A/A_m] \tag{4.14}$$

Note that $P_{\rm avn}$ is negative, indicating that the network is supplying power to the load rather than dissipating power. We can find the maximum of the power delivered to the load by setting the derivative of $-P_{\rm avn}$ with respect to A to zero:

$$\left|\frac{d(-P_{\rm avn})}{dA}\right|_{A=A_o} = \frac{1}{2}R_o[2A_o - 3A_o^2/A_m] = 0 \tag{4.15}$$

The solution is

$$A_o = \frac{2}{3}A_m \tag{4.16}$$

At $A = A_o$, the device input resistance is

$$R_{\rm in}(A_o,\omega) = -R_o\left[1 - \frac{2}{3}\right] = -\frac{1}{3}R_o \tag{4.17}$$

so that in steady state oscillation we want $R_{\rm in} = -R_o/3$ to maximize the output power from the oscillation. Intuitively speaking, as the current amplitude A through the device increases, the device saturates and $|R_{\rm in}|$ will decrease until $R_{\rm in} + R_L = 0$. So, we back off from the saturation point to a smaller value of A. The resulting design rule is that if $R_{\rm in}(0,\omega) = -R_o$ for small signal conditions, then we choose $R_L = R_o/3$.

4.2 Negative Resistance Oscillator Design

We will now look at a design procedure for a transistor oscillator. The goal is to take a transistor and add networks to one of the ports to produce a one port device that is unstable, or in other words, exhibits negative resistance. The one-port procedure developed above can then be used to design a tuning network to complete the oscillator.

If the transistor is not unstable enough for use in an oscillator, an additional step is required to increase the device instability. Generally, we consider a transistor as a two-port device, but if the transistor is too stable, we can treat it as a three-port device and add reactance to one of the ports, to produce a new two-port device which is more unstable.

The design procedure is as follows:

1. Select a transistor and a DC bias point. Generally, you are given the two-port common emitter or common source S-parameters from the manufacturer.
2. *Increasing the instability of the transistor.* If the transistor is unconditionally stable or it is potentially unstable but the unstable region is not large enough, the stability characteristics of the transistor need to be changed. This can generally be done by adding a reactance to one of the device terminals. Commonly, a reactance in the base/gate will work well.
   (a) Compute the 3-port S-parameters for the device from the common emitter parameters (see the section Appendix). Port designations are: 1 = base/gate, 2 = collector/drain, 3 = emitter/source.

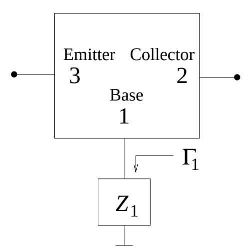

Figure 4.4: Terminating the gate of a transistor with an impedance in order to change its stability characteristics.

   (b) Decide which port will receive the destabilizing reactance. The base/gate is typically used. Assuming that that port 1 sees a reflection coefficient of Γ_1 due to an impedance terminating the base which are trying to design, we want the new two-port S-parameters for ports 2 and 3 to be such that the reflection coefficients are as large as possible. For the three-port S-parameters,

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \tag{4.18}$$

Since $a_1 = \Gamma_1 b_1 \to b_1 = a_1/\Gamma_1$, the first equation in the linear system becomes

$$\frac{a_1}{\Gamma_1} = S_{11}a_1 + S_{12}a_2 + S_{13}a_3 \tag{4.19}$$

$$a_1 = \frac{S_{12}\Gamma_1}{1 - S_{11}\Gamma_1}a_2 + \frac{S_{13}\Gamma_1}{1 - S_{11}\Gamma_1}a_3 \tag{4.20}$$

The remaining equations become

$$b_2 = \left[S_{22} + \frac{S_{21}S_{12}\Gamma_1}{1 - S_{11}\Gamma_1}\right] a_2 + \left[S_{23} + \frac{S_{21}S_{13}\Gamma_1}{1 - S_{11}\Gamma_1}\right] a_3 \tag{4.21}$$

$$b_3 = \left[S_{32} + \frac{S_{31}S_{12}\Gamma_1}{1 - S_{11}\Gamma_1}\right] a_2 + \left[S_{33} + \frac{S_{31}S_{13}\Gamma_1}{1 - S_{11}\Gamma_1}\right] a_3 \tag{4.22}$$

This procedure can of course be performed with the terminating reactance at one of the other ports if needed.

We now have a new common base two-port network with S-parameters S^T, or

$$S^T_{11} = S_{22} + \frac{S_{21}S_{12}\Gamma_1}{1 - S_{11}\Gamma_1} \tag{4.23}$$

$$S^T_{12} = S_{23} + \frac{S_{21}S_{13}\Gamma_1}{1 - S_{11}\Gamma_1} \tag{4.24}$$

$$S^T_{21} = S_{32} + \frac{S_{31}S_{12}\Gamma_1}{1 - S_{11}\Gamma_1} \tag{4.25}$$

$$S^T_{22} = S_{33} + \frac{S_{31}S_{13}\Gamma_1}{1 - S_{11}\Gamma_1} \tag{4.26}$$

(c) We need to find a good value for the reflection coefficient Γ_1 of the base reactance. To make the new network unstable, our ultimate goal is to make $|\Gamma_{\text{in}}| > 1$, where Γ_{in} is the reflection coefficient looking into port 1 of the new 2 port network:

$$\Gamma_{\text{in}} = S^T_{11} + \frac{S^T_{12}S^T_{21}\Gamma_T}{1 - S^T_{22}\Gamma_T} \tag{4.27}$$

To make this reflection coefficient large in magnitude, we will pick a value of Γ_1 that makes $|S^T_{11}|$ large. One approach is to consider the Smith Chart as the S^T_{11} plane and plot the $|\Gamma_1| = 1$ circle (corresponding to a reactive element at the base) on the Smith Chart. Solving the S^T_{11} equation for Γ_1 leads to

$$(1 - S_{11}\Gamma_1)S^T_{11} = S_{22}(1 - S_{11}\Gamma_1) + S_{21}S_{12}\Gamma_1 = S_{22} - \Delta\Gamma_1 \tag{4.28}$$

$$(\Delta - S_{11}S^T_{11})\Gamma_1 = S_{22} - S^T_{11} \tag{4.29}$$

$$\Gamma_1 = \frac{S_{22} - S^T_{11}}{\Delta - S_{11}S^T_{11}} \tag{4.30}$$

Setting the magnitude of this expression to one leads to a circle on the S^T_{11} plane with center and radius given by

$$C_1 = \frac{S_{22} - \Delta S^*_{11}}{1 - |S_{11}|^2} \tag{4.31}$$

$$r_1 = \left|\frac{S_{12}S_{21}}{1 - |S_{11}|^2}\right| \tag{4.32}$$

We choose the point $S^T_{11,\max}$ on this circle for which $|S^T_{11}|$ is a maximum. Using Eq. (4.30) with $S^T_{11} = S^T_{11,\max}$, we can compute the corresponding value of Γ_1 that maximimizes $|S^T_{11}|$. The required base reactance can be found from the reflection coefficient:

$$jX_1 = Z_o\frac{1 + \Gamma_1}{1 - \Gamma_1} \tag{4.33}$$

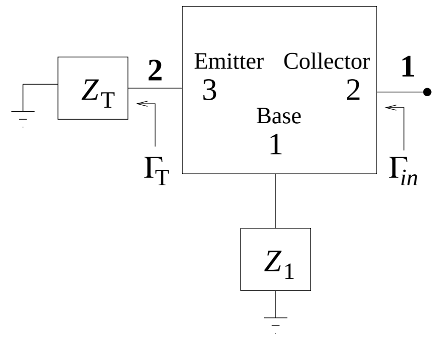

Figure 4.5: Terminating the emitter of the transistor with an impedance in order to achieve a reflection coefficient looking into the collector with magnitude greater than one.

3. *Terminate the 2 port device to make an unstable 1 port device.* We need to design a termination on one of the remaining ports to make the system unstable at the desired frequency. If we choose to put a termination on the emitter (port 2 of the two-port network), then the reflection coefficient at the collector (port 1) is

$$\Gamma_{\text{in}} = S_{11}^T + \frac{S_{12}^T S_{21}^T \Gamma_T}{1 - S_{22}^T \Gamma_T} \tag{4.34}$$

Solving for the reflection coefficient of the port 2 termination gives

$$\Gamma_T = \frac{S_{11}^T - \Gamma_{\text{in}}}{\Delta^T - S_{22}^T \Gamma_{\text{in}}} \tag{4.35}$$

Assuming a purely reactive termination, the $|\Gamma_T| = 1$ circle on the Γ_{in} plane has center and radius

$$C_{\text{in}} = \frac{S_{11}^T - \Delta^T S_{22}^{T*}}{1 - |S_{22}^T|^2} \tag{4.36}$$

$$r_{\text{in}} = \left| \frac{S_{12}^T S_{21}^T}{1 - |S_{22}^T|^2} \right| \tag{4.37}$$

The maximum achievable magnitude for Γ_{in} is

$$\Gamma_{\text{in,max}} = (|C_{\text{in}}| + r_{\text{in}}) \angle C_{\text{in}} \tag{4.38}$$

and we compute the corresponding value of Γ_T from Eq. (4.35). The termination reactance is

$$jX_T = Z_o \frac{1 + \Gamma_T}{1 - \Gamma_T} \tag{4.39}$$

4. Since $|\Gamma_{\text{in}}| > 1$, the real part of the impedance looking into port 1 is negative: $\text{Re}\{Z_{\text{in}}\} < 1$. We now have a one-port negative resistance device, and can use the one-port negative resistance design procedure to determine the load impedance that the collector port should see. We design a tuning network on the collector port to make

$$X_L = -X_{\text{in}} \tag{4.40}$$

$$R_L = |R_{\text{in}}|/3 \tag{4.41}$$

Since the load that the oscillator is driving is a part of the tuning network on the collector side of the transistor, we will take the signal output power from the collector, which is port 1 as a two-port device (or port 2 as a three-port device).

It is also possible to take the power from the emitter (port 2), so that the load is part of the termination network at the emitter instead of the tuning network at the collector. The termination and tuning networks can also be swapped between the emitter and collector.

If the transistor is already potentially unstable and it is not necessary to destabilize it further, step 2 can be skipped.

Section Appendix: Three Port S-Parameters

Suppose we have a three port device which has terminal three connected to ground. The two-port S-parameters for this configuration are specified as

$$[S^E] = \begin{bmatrix} S_{11}^E & S_{12}^E \\ S_{21}^E & S_{22}^E \end{bmatrix}.$$

The corresponding three-port S-parameters may be computed using

$$S_{\text{sum}} = \sum_{i=1,2} \sum_{j=1,2} S_{ij}^E \tag{4.42}$$

$$S_{33} = \frac{S_{\text{sum}}}{4 - S_{\text{sum}}} \tag{4.43}$$

$$S_{32} = \frac{1 + S_{33}}{2} \left(1 - S_{12}^E - S_{22}^E\right) \tag{4.44}$$

$$S_{23} = \frac{1 + S_{33}}{2} \left(1 - S_{21}^E - S_{22}^E\right) \tag{4.45}$$

$$S_{22} = S_{22}^E + \frac{S_{23} S_{32}}{1 + S_{33}} \tag{4.46}$$

$$S_{13} = 1 - S_{23} - S_{33} \tag{4.47}$$

$$S_{31} = 1 - S_{33} - S_{32} \tag{4.48}$$

$$S_{12} = 1 - S_{22} - S_{32} \tag{4.49}$$

$$S_{21} = 1 - S_{22} - S_{23} \tag{4.50}$$

$$S_{11} = 1 - S_{21} - S_{31} \tag{4.51}$$