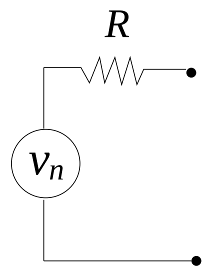

Figure 3.3: Equivalent circuit of a noise source.

We can replace any noisy (warm) resistor with a Thevenin equivalent of a noise source and an ideal, noiseless resistor (Fig. 3.3). If we connect this equivalent circuit to a bandpass filter with bandwidth B Hz and then to a second ideal resistor R (where the resistance of the load is chosedn for maximum power transfer), the noise power delivered to the load is

$$P_n = \left(\frac{\overline{v}_n}{2R}\right)^2 R = \frac{\overline{v}_n^2}{4R} \tag{3.83}$$

Note that we do not have another factor of two in the denominator as we would for phasor voltages since $\overline{v}_n$ is already an RMS quantity. Using our expression for $\overline{v}_n$ leads to

$$P_n = \frac{4k_B TBR}{4R} = k_B TB \tag{3.84}$$

This result is very often used for other noise sources than resistors. The noise source may not even be at a physical temperature equal to T, in which case T in (3.84) becomes an equivalent noise temperature.

When working with microwave signals, it is often convenient to use units of dBm, which means power expressed in decibels relative to 1 milliwatt (dBm is $10\log_{10}$[Power(mW)]). For a resistor at room temperature (approximately $T = 290$ K), $10\log_{10} k_B T = -174$ dBm/Hz. In order to go from this quantity, which measures the amount of noise power in a 1 Hz bandwidth, we multiply by the system bandwidth, or add $10\log_{10} B$ in dB to find the total in-band noise power.

3.5.1 Noise Figure

A key measure of system performance is signal-to-noise ratio (SNR):

$$\text{SNR} = \frac{S}{N} = \frac{\text{Signal Power}}{\text{Noise Power}} \tag{3.85}$$

A high SNR means that it is easy to recognize the signal, and a low SNR means that the signal is obscured by noise.

Amplifiers, lossy transmission lines, mixers, and almost any other component of a microwave system add noise to the signal. An ideal component does not add any noise, so the SNR at the output is the same as the SNR at the input. But for a non-ideal component, the output SNR is always less than the input SNR.

Noise figure is a measure of the degradation in signal-to-noise ratio (SNR) as a signal passes through a system component. The definition of noise figure (F) is the ratio of the total available noise power at the amplifier output to the available noise power at the output due to the input noise only:

$$F = \frac{\text{Output noise power}}{\text{Ideal output noise power = Gain} \times \text{input noise power}} \geq 1 \tag{3.86}$$

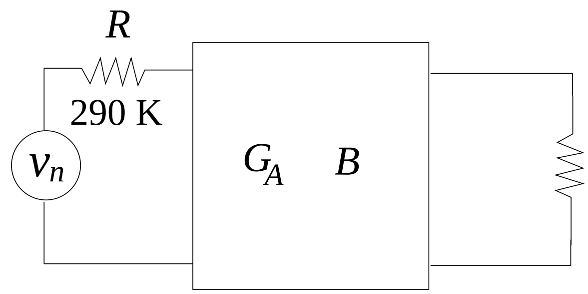

Figure 3.4: Noisy amplifier.

For an ideal component, $F = 1$. The gain used in this expression is the available gain

$$G_A = \frac{P_{\text{avn}}}{P_{\text{avs}}} = \frac{S_o}{S_i} \tag{3.87}$$

It can be seen that noise figure is also equal to the ratio of the input SNR to the output SNR:

$$F = \frac{N_o}{N_i G_A} = \frac{N_o}{N_i S_o/S_i} = \frac{S_i/N_i}{S_o/N_o} = \frac{\text{SNR}_{\text{in}}}{\text{SNR}_{\text{out}}} \tag{3.88}$$

We can also write

$$F = \frac{G_A N_i + P_n}{G_A N_i} = 1 + \frac{P_n}{G_A N_i} \tag{3.89}$$

where P_n is the extra noise power at the output introduced by the component. As a convention, we assume that the input noise corresponds to room temperature, so that $N_i = k_B T_0 B$ with $T_0 = 290\,\text{K}$. Since noise figure is a dimensionless quantity, it is often expressed in dB.

3.5.2 Equivalent Noise Temperature

We can also express the "noisiness" of a component in terms of an equivalent noise temperature using $P = k_B T B$. If we consider an ideal, noiseless component with a warm resistor at the input, then the equivalent temperature T_e is defined to be the temperature of the resistor such that it supplies the same noise as the non-ideal component, so that

$$P_n = G_A k_B T_e B \tag{3.90}$$

Using this in Eq. (3.89) together with $N_i = k_B T_0 B$ leads to

$$F = 1 + \frac{T_e}{T_0} \tag{3.91}$$

Equivalent temperature is often used for very low noise figure devices.

3.5.3 Lossy Components

A lossy system component such as a length of lossy transmission line leads to a degradation in SNR. The basic principle for determining the noise figure of a lossy component is to realize that the noise power at the output of the component must be the same as the noise power at the input (thermal equilibrium), so that

$$G N_i + P_n = N_i \tag{3.92}$$

Solving for the equivalent additional power at the input gives $P_n = N_i(1-G)$. The noise figure is then

$$F = 1 + \frac{P_n}{GN_i} = 1 + \frac{N_i(1-G)}{GN_i} = \frac{1}{G} = L \tag{3.93}$$

where L is the power loss of the device. Thus, the noise figure is the same as the loss.

3.5.4 Cascaded Networks

If we have two stages in a system,

$$N_o = G_{A2}N_{o1} + P_{n2} = G_{A2}(G_{A1}N_i + P_{n1}) + P_{n2} \tag{3.94}$$

$$F = \frac{G_{A2}(G_{A1}N_i + P_{n1}) + P_{n2}}{N_i G_{A1} G_{A2}} = 1 + \frac{P_{n1}}{N_i G_{A1}} + \frac{P_{n2}}{N_i G_{A1} G_{A2}} \tag{3.95}$$

In terms of the noise figures of the two stages,

$$F_1 = 1 + \frac{P_{n1}}{N_i G_{A1}} \tag{3.96}$$

$$F_2 = 1 + \frac{P_{n2}}{N_i G_{A2}} \tag{3.97}$$

the noise figure of the system is

$$F = F_1 + \frac{F_2 - 1}{G_{A1}} \tag{3.98}$$

The noise figure of the second state is divided by the gain of the first stage. We can see that the first stage is most critical in determining the noise figure of the system. The idea is that we want to boost the signal as much as possible early in the system while adding as little possible noise so that the signal is larger than noise added by subsequent components in the system. For a receive antenna, for example, we want to have an amplifier with high gain and a noise figure close to unity before a long length of lossy coaxial cable.

3.6 Low Noise Amplifiers

For an amplifier, it can be shown that

$$F = F_{\min} + \frac{R_N}{G_s}|Y_s - Y_{\text{opt}}|^2 \tag{3.99}$$

where

$$\begin{aligned}
Y_s &= G_s + jB_s = \text{source admittance} \\
Y_{\text{opt}} &= \text{optimum source admittance resulting in minimum noise figure} \\
F_{\min} &= \text{minimum noise figure} \\
R_N &= \text{equivalent noise resistance of the transistor}
\end{aligned}$$

Y_{opt}, $F_{\min}$, and R_N are noise parameters for the transistor, and would typically be measured or included in a spec sheet for the transistor.

We want to put Eq. (3.99) in terms of reflection coefficients rather than admittances. Using

$$Y_s = \frac{1}{Z_o}\frac{1-\Gamma_s}{1+\Gamma_s}, \qquad Y_{\text{opt}} = \frac{1}{Z_o}\frac{1-\Gamma_{\text{opt}}}{1+\Gamma_{\text{opt}}} \tag{3.100}$$

the magnitude squared term in Eq. (3.99) becomes

$$\begin{aligned}
|Y_s - Y_{opt}|^2 &= \frac{1}{Z_o^2}\left|\frac{1-\Gamma_s}{1+\Gamma_s} - \frac{1-\Gamma_{\text{opt}}}{1+\Gamma_{\text{opt}}}\right|^2 \\
&= \frac{1}{Z_o^2}\left|\frac{1-\Gamma_s+\Gamma_{\text{opt}}-\Gamma_s\Gamma_{\text{opt}}-1-\Gamma_s+\Gamma_{\text{opt}}+\Gamma_s\Gamma_{\text{opt}}}{(1+\Gamma_s)(1+\Gamma_{\text{opt}})}\right|^2 \\
&= \frac{1}{Z_o^2}\left|\frac{-2\Gamma_s+2\Gamma_{\text{opt}}}{(1+\Gamma_s)(1+\Gamma_{\text{opt}})}\right|^2 \\
&= \frac{4}{Z_o^2}\left|\frac{\Gamma_s-\Gamma_{\text{opt}}}{(1+\Gamma_s)(1+\Gamma_{\text{opt}})}\right|^2
\end{aligned} \tag{3.101}$$

The source conductance is

$$\begin{aligned}
G_s &= \text{Re}\{Y_s\} = \frac{1}{2}(Y_s + Y_s^*) \\
&= \frac{1}{2Z_o}\left[\frac{1-\Gamma_s}{1+\Gamma_s} + \frac{1-\Gamma_s^*}{1+\Gamma_s^*}\right] \\
&= \frac{1}{2Z_o}\left[\frac{1-\Gamma_s+\Gamma_s^*-|\Gamma_s|^2+1+\Gamma_s-\Gamma_s^*-|\Gamma_s|^2}{|1+\Gamma_s|^2}\right] \\
&= \frac{1}{Z_o}\frac{1-|\Gamma_s|^2}{|1+\Gamma_s|^2}
\end{aligned} \tag{3.102}$$

Using these expressions, the amplifier noise figure becomes

$$\begin{aligned}
F &= F_{\min} + R_N Z_o \frac{1-|\Gamma_s|^2}{|1+\Gamma_s|^2}\frac{4}{Z_o^2}\left|\frac{\Gamma_s-\Gamma_{\text{opt}}}{(1+\Gamma_s)(1+\Gamma_{\text{opt}})}\right|^2 \\
&= F_{\min} + \frac{4R_N}{Z_o}\frac{|\Gamma_s-\Gamma_{\text{opt}}|^2}{(1-|\Gamma_s|^2)|1+\Gamma_{\text{opt}}|^2}
\end{aligned} \tag{3.103}$$

Now, what we would like is to know the values of Γ_s that give a fixed noise figure. To do this, we first define a noise figure parameter N, which consists of all the factors in (3.103) that do not depend on Γ_s:

$$N = \frac{F - F_{\min}}{4R_N/Z_o}|1 + \Gamma_{\text{opt}}|^2 \tag{3.104}$$

We do this to isolate the terms containing Γ_s, and lump the rest into N. Therefore,

$$\begin{aligned}
(\Gamma_s - \Gamma_{\text{opt}})(\Gamma_s^* - \Gamma_{\text{opt}}^*) &= N(1 - \Gamma_s\Gamma_s^*) \\
|\Gamma_s|^2 - \Gamma_s\Gamma_{\text{opt}}^* - \Gamma_s^*\Gamma_{\text{opt}} + |\Gamma_{\text{opt}}|^2 &= N(1 - |\Gamma_s|^2) \\
|\Gamma_s|^2 - \Gamma_s\frac{\Gamma_{\text{opt}}^*}{1+N} - \Gamma_s^*\frac{\Gamma_{\text{opt}}}{1+N} &= \frac{N - |\Gamma_{\text{opt}}|^2}{1+N} \qquad (3.105)
\end{aligned}$$

Once again, we see this is a circle in the complex plane, with center and radius given by

$$C_F = \frac{\Gamma_{\text{opt}}}{N+1} \tag{3.106}$$

$$r_F = \frac{\sqrt{N(N+1-|\Gamma_{\text{opt}}|^2)}}{N+1} \tag{3.107}$$

Using these expressions, we can draw gain, stability, and noise figure circles on the Γ_s Smith chart and pick a value of Γ_s to achieve multiple specifications.

3.7 Dynamic Range Issues for Amplifiers

There are a few things we need to understand about the power operation of amplifiers.

1. *1 dB Compression Point:* This is defined as the output power at which the gain has dropped 1 dB from its low-power value. Note that the slope of the output versus input power curve is 1 dB/dB. We often denote this point as $P_{1\text{dB}}$.

2. *Dynamic Range:* Range of input that can be detected by the receiver without appreciable distortion. Consider an amplifier with a noise figure F:

$$F = \frac{N_o}{G_A N_i} = \frac{N_o}{G_A k_B T B} \tag{3.108}$$

$$N_o = F G_A k_B T B \tag{3.109}$$

If the minimum detectable signal for the receiver output (denoted as $S_{o,\text{mds}}$) is X dB above the noise floor, then

$$S_{o,\text{mds}} = -174 \text{ dBm } + 10\log_{10} B + F_{\text{dB}} + X + G_{A,\text{dB}} \tag{3.110}$$

where we have used that $10\log_{10}(10^3 k_B T) = -174$ dBm at $T = 290$ K. The dynamic range is then the difference between the 1 dB compression point $P_{1\text{dB}}$ and $S_{o,\text{mds}}$, or

$$DR = P_{1\text{dB}} - S_{o,\text{mds}} = P_{1\text{dB}} + 174 \text{ dBm } - 10\log_{10} B - F_{\text{dB}} - X - G_{A,\text{dB}} \tag{3.111}$$

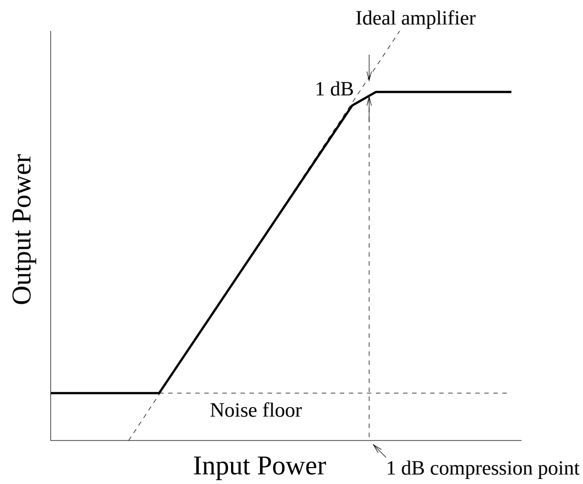

Figure 3.5: Dynamic range of an amplifier.

3. *Third Order Intercept (TOI, TOIP, IP$_3$):* Consider a two-tone test where the input signal is

$$v(t) = A\cos(2\pi f_1 t) + A\cos(2\pi f_2 t) \tag{3.112}$$

where $|f_1 - f_2|$ 5 to 10 MHz. The output frequencies will be of the form

$$f_o = mf_1 + nf_2 \tag{3.113}$$

where m and n are integers. The order of the intermodulation product (IP) is given by $|m| + |n|$.

Note that $2f_1 - f_2$ and $2f_2 - f_1$ will be inside the communication band. The third order intercept point P_{IP} is defines as the output power at which the third order IP power intersects the linear power (assuming no gain compression or saturation occurs). The slope of the third order intermodulation product output power versus input power is 3 dB/dB.

4. *Spurious Free Dynamic Range:* To compute this dynamic range, we continue to use $S_{o,\mathrm{mds}}$ as the lower bound. However, for the upper bound, we take the output power (in the fundamental signal) at which the third order intermodulation product output power reaches $S_{o,\mathrm{mds}}$.