Chapter 4

Oscillators

There are several types of sources of microwave signals:

- *Black-body radiation.* All materials give off a small amount of microwave radiation due to black-body or thermal radiation. This effect is used in passive remote sensing and receiver calibration.
- *Microwave tubes.* These sources are typically used for very high power applications.
- *Diodes.* A diode source converts DC into microwave energy by making use of a negative resistance voltage-current characteristic (one-port oscillators).
- *Transistors.* Transistor microwave oscillators are similar in principle to low frequency oscillators—amplifiers with feedback (two-port oscillators).

4.1 Oscillator Basics

In studying the stability of an amplifier design, we recognized that we can have a transistor with an input or output reflection coefficients have a magnitude greater than unity. The question is: how do we use this fact to form an oscillator?

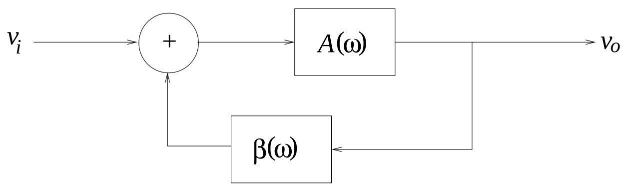

Figure 4.1: Block diagram of a simple feedback network.

By analyzing a simple feedback network, we can find the conditions necessary for oscillation. From the block diagram in Fig. 4.1, the output signal is

$$v_o = A(\omega)[v_i + \beta(\omega)v_o] \tag{4.1}$$

Rearranding this expression gives the transfer function of the system:

$$\frac{v_o}{v_i} = \frac{A(\omega)}{1 - \beta(\omega)A(\omega)} \tag{4.2}$$

For oscillation to occur, we want an output v_o with no input v_i ($v_o/v_i \to \infty$). This leads to the condition

$$\beta(\omega)A(\omega) = 1 \tag{4.3}$$

We see that the loop gain must have: 1) unity magnitude and 2) $2\pi n$ phase, where n is an integer.

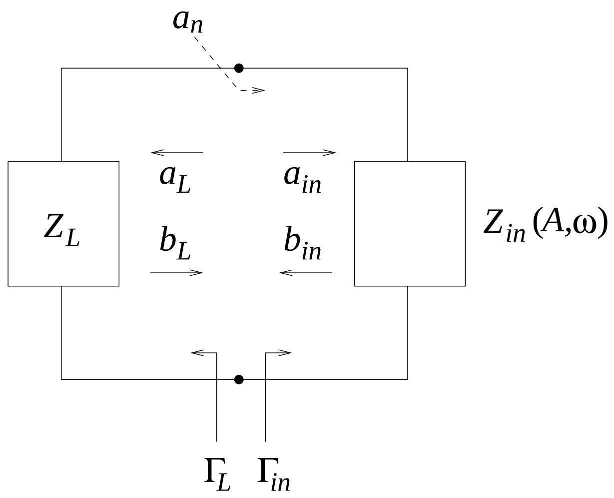

Figure 4.2: Oscillator network with load on the left and active (nonlinear) device on the right. Because the device is nonlinear, its input impedance Z_{in} is a function of the current amplitude A and frequency ω.

Now, let us look at the same concept from a microwave network point of view, as shown in Fig. 4.2. We assume that some noise signal a_n due to thermal or another type of noise is input as an additional forward wave into the active device. The total forward wave into the device is

$$\begin{aligned} a_{\text{in}} &= a_n + \Gamma_{\text{in}}\Gamma_L a_{\text{in}} \\ &= \frac{a_n}{1 - \Gamma_{\text{in}}\Gamma_L} \end{aligned} \tag{4.4}$$

The forward wave into the load is

$$a_L = \Gamma_{\text{in}} a_{\text{in}} = \frac{a_n \Gamma_{\text{in}}}{1 - \Gamma_{\text{in}}\Gamma_L} \tag{4.5}$$

For oscillation, we must have

$$\Gamma_{\text{in}}\Gamma_L = 1 \tag{4.6}$$

We now want to change this to a condition on the load and device impedances:

$$\begin{aligned} \Gamma_{\text{in}}(A,\omega)\Gamma_L(\omega) &= 1 \\ \left[\frac{Z_{\text{in}}(A,\omega) - Z_o}{Z_{\text{in}}(A,\omega) + Z_o}\right]\left[\frac{Z_L(\omega) - Z_o}{Z_L(\omega) + Z_o}\right] &= 1 \\ Z_{\text{in}}Z_L - Z_o(Z_{\text{in}} + Z_L) + Z_o^2 &= Z_{\text{in}}Z_L + Z_o(Z_{\text{in}} + Z_L) + Z_o^2 \\ Z_{\text{in}} + Z_L &= 0 \end{aligned} \tag{4.7}$$

where A is the signal amplitude. Breaking up the impedances into real and reactive parts leads to the conditions

$$R_{\text{in}}(A,\omega) + R_L(\omega) = 0 \tag{4.8}$$
$$X_{\text{in}}(A,\omega) + X_L(\omega) = 0 \tag{4.9}$$

For the reactive part, we choose $X_L(\omega) = -X_{\text{in}}(A,\omega)$. Now, the question is how to pick $R_L(\omega)$ so that oscillation starts up properly and to provide high output power.

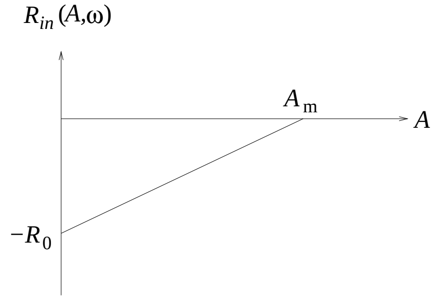

Figure 4.3: Device resistance as a function of current amplitude A.

Let us assume that $R_{\text{in}}(A,\omega)$ is linear in A. (Does this mean that the device is linear?) Let us denote the $A = 0$ intercept as $R_{\text{in}}(0,\omega) = -R_o$, as shown in Fig. 4.3. The input resistance can be expressed as

$$R_{\text{in}}(A,\omega) = -R_o\,[1 - A/A_m] \tag{4.10}$$

where A_m is the output signal level at which $R_{in}(A_m,\omega) = 0$. The power available to the load is

$$P_{\text{avn}} = \frac{1}{2}\text{Re}\{VI^*\} \tag{4.11}$$
$$= \frac{1}{2}|I|^2\text{Re}\{Z_{\text{in}}(A,\omega)\} \tag{4.12}$$
$$= \frac{1}{2}|I|^2 R_{\text{in}}(A,\omega) \tag{4.13}$$
$$= -\frac{1}{2}A^2 R_o\,[1 - A/A_m] \tag{4.14}$$

Note that P_{avn} is negative, indicating that the network is supplying power to the load rather than dissipating power. We can find the maximum of the power delivered to the load by setting the derivative of $-P_{\text{avn}}$ with respect to A to zero:

$$\left|\frac{d(-P_{\text{avn}})}{dA}\right|_{A=A_o} = \frac{1}{2}R_o[2A_o - 3A_o^2/A_m] = 0 \tag{4.15}$$

The solution is

$$A_o = \frac{2}{3}A_m \tag{4.16}$$

At $A = A_o$, the device input resistance is

$$R_{\text{in}}(A_o,\omega) = -R_o\left[1 - \frac{2}{3}\right] = -\frac{1}{3}R_o \tag{4.17}$$

so that in steady state oscillation we want $R_{\text{in}} = -R_o/3$ to maximize the output power from the oscillation. Intuitively speaking, as the current amplitude A through the device increases, the device saturates and $|R_{\text{in}}|$ will decrease until $R_{\text{in}} + R_L = 0$. So, we back off from the saturation point to a smaller value of A. The resulting design rule is that if $R_{\text{in}}(0,\omega) = -R_o$ for small signal conditions, then we choose $R_L = R_o/3$.

4.2 Negative Resistance Oscillator Design

We will now look at a design procedure for a transistor oscillator. The goal is to take a transistor and add networks to one of the ports to produce a one port device that is unstable, or in other words, exhibits negative resistance. The one-port procedure developed above can then be used to design a tuning network to complete the oscillator.

If the transistor is not unstable enough for use in an oscillator, an additional step is required to increase the device instability. Generally, we consider a transistor as a two-port device, but if the transistor is too stable, we can treat it as a three-port device and add reactance to one of the ports, to produce a new two-port device which is more unstable.

The design procedure is as follows:

1. Select a transistor and a DC bias point. Generally, you are given the two-port common emitter or common source S-parameters from the manufacturer.
2. *Increasing the instability of the transistor.* If the transistor is unconditionally stable or it is potentially unstable but the unstable region is not large enough, the stability characteristics of the transistor need to be changed. This can generally be done by adding a reactance to one of the device terminals. Commonly, a reactance in the base/gate will work well.

   (a) Compute the 3-port S-parameters for the device from the common emitter parameters (see the section Appendix). Port designations are: 1 = base/gate, 2 = collector/drain, 3 = emitter/source.

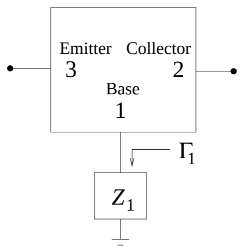

Figure 4.4: Terminating the gate of a transistor with an impedance in order to change its stability characteristics.

   (b) Decide which port will receive the destabilizing reactance. The base/gate is typically used. Assuming that that port 1 sees a reflection coefficient of Γ_1 due to an impedance terminating the base which are trying to design, we want the new two-port S-parameters for ports 2 and 3 to be such that the reflection coefficients are as large as possible. For the three-port S-parameters,

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \tag{4.18}$$

   Since $a_1 = \Gamma_1 b_1 \to b_1 = a_1/\Gamma_1$, the first equation in the linear system becomes

$$\frac{a_1}{\Gamma_1} = S_{11}a_1 + S_{12}a_2 + S_{13}a_3 \tag{4.19}$$

$$a_1 = \frac{S_{12}\Gamma_1}{1 - S_{11}\Gamma_1}a_2 + \frac{S_{13}\Gamma_1}{1 - S_{11}\Gamma_1}a_3 \tag{4.20}$$

The remaining equations become

$$b_2 = \left[S_{22} + \frac{S_{21}S_{12}\Gamma_1}{1 - S_{11}\Gamma_1}\right] a_2 + \left[S_{23} + \frac{S_{21}S_{13}\Gamma_1}{1 - S_{11}\Gamma_1}\right] a_3 \quad (4.21)$$

$$b_3 = \left[S_{32} + \frac{S_{31}S_{12}\Gamma_1}{1 - S_{11}\Gamma_1}\right] a_2 + \left[S_{33} + \frac{S_{31}S_{13}\Gamma_1}{1 - S_{11}\Gamma_1}\right] a_3 \quad (4.22)$$

This procedure can of course be performed with the terminating reactance at one of the other ports if needed.

We now have a new common base two-port network with S-parameters S^T, or

$$S_{11}^T = S_{22} + \frac{S_{21}S_{12}\Gamma_1}{1 - S_{11}\Gamma_1} \quad (4.23)$$

$$S_{12}^T = S_{23} + \frac{S_{21}S_{13}\Gamma_1}{1 - S_{11}\Gamma_1} \quad (4.24)$$

$$S_{21}^T = S_{32} + \frac{S_{31}S_{12}\Gamma_1}{1 - S_{11}\Gamma_1} \quad (4.25)$$

$$S_{22}^T = S_{33} + \frac{S_{31}S_{13}\Gamma_1}{1 - S_{11}\Gamma_1} \quad (4.26)$$

(c) We need to find a good value for the reflection coefficient Γ_1 of the base reactance. To make the new network unstable, our ultimate goal is to make $|\Gamma_{\text{in}}| > 1$, where Γ_{in} is the reflection coefficient looking into port 1 of the new 2 port network:

$$\Gamma_{\text{in}} = S_{11}^T + \frac{S_{12}^T S_{21}^T \Gamma_T}{1 - S_{22}^T \Gamma_T} \quad (4.27)$$

To make this reflection coefficient large in magnitude, we will pick a value of Γ_1 that makes $|S_{11}^T|$ large. One approach is to consider the Smith Chart as the S_{11}^T plane and plot the $|\Gamma_1| = 1$ circle (corresponding to a reactive element at the base) on the Smith Chart. Solving the S_{11}^T equation for Γ_1 leads to

$$(1 - S_{11}\Gamma_1)S_{11}^T = S_{22}(1 - S_{11}\Gamma_1) + S_{21}S_{12}\Gamma_1 = S_{22} - \Delta\Gamma_1 \quad (4.28)$$

$$(\Delta - S_{11}S_{11}^T)\Gamma_1 = S_{22} - S_{11}^T \quad (4.29)$$

$$\Gamma_1 = \frac{S_{22} - S_{11}^T}{\Delta - S_{11}S_{11}^T} \quad (4.30)$$

Setting the magnitude of this expression to one leads to a circle on the S_{11}^T plane with center and radius given by

$$C_1 = \frac{S_{22} - \Delta S_{11}^*}{1 - |S_{11}|^2} \quad (4.31)$$

$$r_1 = \left|\frac{S_{12}S_{21}}{1 - |S_{11}|^2}\right| \quad (4.32)$$

We choose the point $S_{11,\max}^T$ on this circle for which $|S_{11}^T|$ is a maximum. Using Eq. (4.30) with $S_{11}^T = S_{11,\max}^T$, we can compute the corresponding value of Γ_1 that maximimizes $|S_{11}^T|$. The required base reactance can be found from the reflection coefficient:

$$jX_1 = Z_o \frac{1 + \Gamma_1}{1 - \Gamma_1} \quad (4.33)$$

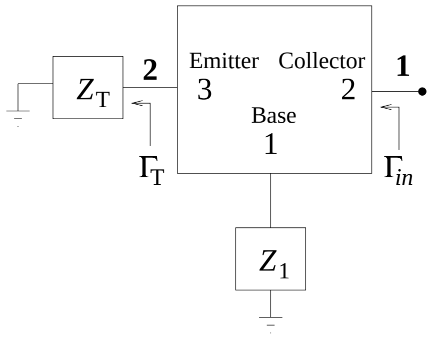

Figure 4.5: Terminating the emitter of the transistor with an impedance in order to achieve a reflection coefficient looking into the collector with magnitude greater than one.

3. *Terminate the 2 port device to make an unstable 1 port device.* We need to design a termination on one of the remaining ports to make the system unstable at the desired frequency. If we choose to put a termination on the emitter (port 2 of the two-port network), then the reflection coefficient at the collector (port 1) is

$$\Gamma_{\text{in}} = S_{11}^T + \frac{S_{12}^T S_{21}^T \Gamma_T}{1 - S_{22}^T \Gamma_T} \tag{4.34}$$

Solving for the reflection coefficient of the port 2 termination gives

$$\Gamma_T = \frac{S_{11}^T - \Gamma_{\text{in}}}{\Delta^T - S_{22}^T \Gamma_{\text{in}}} \tag{4.35}$$

Assuming a purely reactive termination, the $|\Gamma_T| = 1$ circle on the Γ_{in} plane has center and radius

$$C_{\text{in}} = \frac{S_{11}^T - \Delta^T S_{22}^{T*}}{1 - |S_{22}^T|^2} \tag{4.36}$$

$$r_{\text{in}} = \left| \frac{S_{12}^T S_{21}^T}{1 - |S_{22}^T|^2} \right| \tag{4.37}$$

The maximum achievable magnitude for Γ_{in} is

$$\Gamma_{\text{in,max}} = (|C_{\text{in}}| + r_{\text{in}}) \angle C_{\text{in}} \tag{4.38}$$

and we compute the corresponding value of Γ_T from Eq. (4.35). The termination reactance is

$$jX_T = Z_o \frac{1 + \Gamma_T}{1 - \Gamma_T} \tag{4.39}$$

4. Since $|\Gamma_{\text{in}}| > 1$, the real part of the impedance looking into port 1 is negative: Re$\{Z_{\text{in}}\} < 1$. We now have a one-port negative resistance device, and can use the one-port negative resistance design procedure to determine the load impedance that the collector port should see. We design a tuning network on the collector port to make

$$X_L = -X_{\text{in}} \tag{4.40}$$

$$R_L = |R_{\text{in}}|/3 \tag{4.41}$$

Since the load that the oscillator is driving is a part of the tuning network on the collector side of the transistor, we will take the signal output power from the collector, which is port 1 as a two-port device (or port 2 as a three-port device).

It is also possible to take the power from the emitter (port 2), so that the load is part of the termination network at the emitter instead of the tuning network at the collector. The termination and tuning networks can also be swapped between the emitter and collector.

If the transistor is already potentially unstable and it is not necessary to destabilize it further, step 2 can be skipped.

Section Appendix: Three Port S-Parameters

Suppose we have a three port device which has terminal three connected to ground. The two-port S-parameters for this configuration are specified as

$$[S^E] = \begin{bmatrix} S_{11}^E & S_{12}^E \\ S_{21}^E & S_{22}^E \end{bmatrix}.$$

The corresponding three-port S-parameters may be computed using

$$S_{\text{sum}} = \sum_{i=1,2} \sum_{j=1,2} S_{ij}^E \tag{4.42}$$

$$S_{33} = \frac{S_{\text{sum}}}{4 - S_{\text{sum}}} \tag{4.43}$$

$$S_{32} = \frac{1 + S_{33}}{2} \left(1 - S_{12}^E - S_{22}^E\right) \tag{4.44}$$

$$S_{23} = \frac{1 + S_{33}}{2} \left(1 - S_{21}^E - S_{22}^E\right) \tag{4.45}$$

$$S_{22} = S_{22}^E + \frac{S_{23} S_{32}}{1 + S_{33}} \tag{4.46}$$

$$S_{13} = 1 - S_{23} - S_{33} \tag{4.47}$$

$$S_{31} = 1 - S_{33} - S_{32} \tag{4.48}$$

$$S_{12} = 1 - S_{22} - S_{32} \tag{4.49}$$

$$S_{21} = 1 - S_{22} - S_{23} \tag{4.50}$$

$$S_{11} = 1 - S_{21} - S_{31} \tag{4.51}$$

Chapter 5

Mixers

A mixer is a device that multiplies two signals. In RF and microwave design, mixers are used to upconvert or downconvert a signal in frequency. If a narrowband signal centered around a carrier frequency is mixed with a sinusoidal signal at the same frequency as the carrier, one of the mixing products that appears at the output is the narrowband signal centered at DC. Often, instead of mixing a signal all the way to DC, it is common to mix to an intermediate frequency (IF) such as 70 MHz or 140 MHz that is much smaller than the carrier frequency. Further processing and detection of the information carried in the signal can then be done at the lower frequency with much simpler circuits than would be required at the original carrier frequency.

RF mixers can also be used for modulation/demodulation, although this is uncommon since most modern communication signals use complex modulations which would require two mixers with nearly identical phase performance. Since this is difficult to construct at high frequencies, modulation is usually done at a lower IF frequency and then the signal is upconverted to a center frequency in the microwave band.

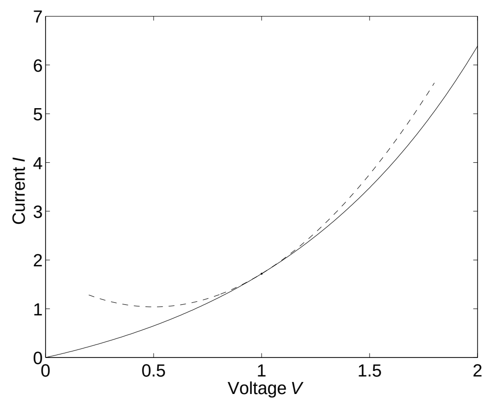

Figure 5.1: Nonlinear diode current/voltage characteristic. Solid line: Eq. (5.1) with $\alpha = I_s = 1$. Dashed line: second order approximation using the first three terms of Eq. (5.2).

One way to perform mixing is take two signals of similar strength, add them together, and then run them through a diode. To see how this works mathematically, consider that a diode has a voltage-current relationship of

$$I(V) = I_s(e^{\alpha V} - 1) \tag{5.1}$$

If the voltage is written as $V = V_o + v$, where V_o is a DC bias voltage and v is a small AC term, then we can write a Taylor series of the current as

$$I(V_o + v) = I(V_o) + v \left.\frac{dI}{dV}\right|_{V=V_o} + \frac{v^2}{2} \left.\frac{d^2I}{dV^2}\right|_{V=V_o} + \cdots \tag{5.2}$$

where we can neglect higher order terms as long as v is small. Defining $dI/dV = 1/R_j$ at $V = V_o$, where R_j is the small-signal junction resistance, then $d^2I/dV^2 = \alpha/R_j$ at $V = V_o$. If $v = v_1 + v_2$, then the second-order term in the Taylor series produces

$$\frac{v^2}{2} \left.\frac{d^2I}{dV^2}\right|_{V=V_o} = \frac{\alpha}{2R_j}(v_1^2 + 2v_1v_2 + v_2^2) \tag{5.3}$$

The middle term performs the multiplication we desire. So, all we have to do is match the diode to the feedline in order to maximize the voltage across the diode junction and thereby maximize the signal strength of the multiplied signal.

This nonlinear diode relationship also indicates how to do power detection. We often need to do power sampling in a wireless communication system, for example, when the gain of the receiver must be be automatically varied depending on the received signal strength. If the input voltage is $v(t) = A\cos\omega t$, then the squaring operation leads to

$$\frac{v^2}{2} \left.\frac{d^2I}{dV^2}\right|_{V=V_o} = \frac{\alpha A^2}{4R_j}\left[1 + \cos(2\omega t)\right] \tag{5.4}$$

Filtering out the term at $2\omega t$ leaves a DC term that is proportional to the received signal power (voltage squared).

5.1 Switching (Sampling) Mixers

The diode mixer described above is relatively straightforward to design and build, and is used in some cases for special purpose designs, but most commercial mixers use an different approach based on switching. Switching mixers are not as intuitive as nonlinear mixing, but can be readily analyzed using Fourier analysis.

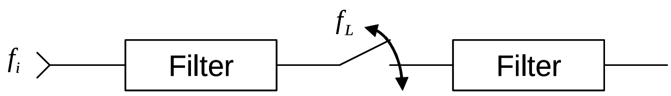

Figure 5.2: A simple sampling mixer configuration.

Consider a system with a high-speed switch that samples an input signal at frequency f_i at a sample rate of f_L. The system is shown in Fig. 5.2. Assume that the switch is an ideal sampling device for now, which means that it is closed only instantaneously. The sampling function is therefore a sequence of delta functions. The spectrum of the sampling function will also be a sequence of delta functions, as shown in Fig. 5.3.

Since we are multiplying the signal by $v_L(t)$, we are convolving in the frequency domain. This will replicate the signal spectrum at f_L intervals in the frequency domain. We can then filter out images of the spectrum that we do not want. Note that we call v_L the *Local Oscillator (LO)* signal. The high frequency signal is typically called the *Radio Frequency (RF)* signal, and the low frequency signal the *Intermediate Frequency*

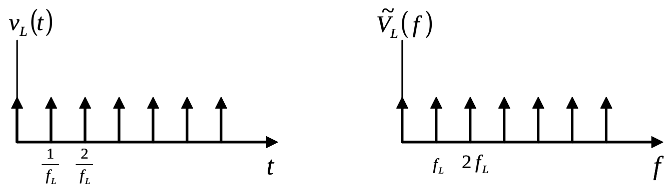

Figure 5.3: Sampling voltage and spectrum.

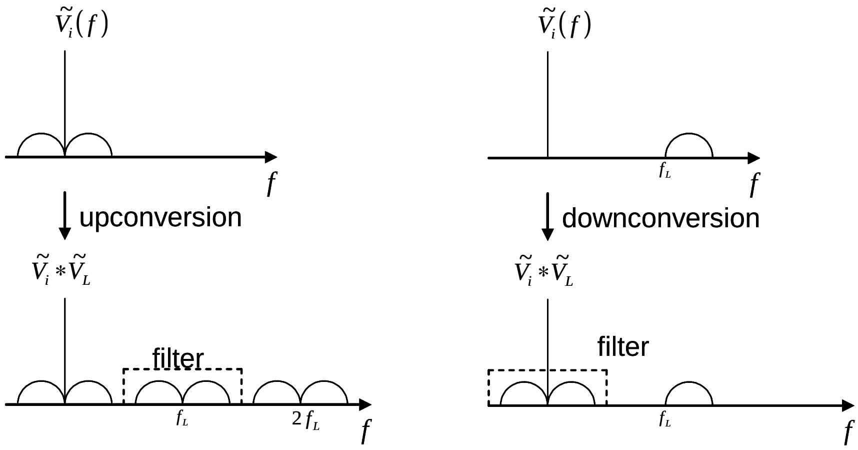

Figure 5.4: Sampling mixer up- and downconversion of a signal.

(IF) signal. If the RF is the input, IF is the output (downconversion). If the IF is the input, the RF is the output (upconversion). Upconversion and downconversion are represented in the spectral domain in Fig. 5.4.

Now, suppose $v_L(t)$ is not a train of impulses, but rather some periodic function with a period of $T_L = 1/f_L$. $\tilde{V}_L(f)$ will still be impulses at nf_L. However, they will not be equal amplitude, since the spectrum of the sampling function decays with frequency. This is what we obtain using a diode as the switching device (Fig. 5.5).

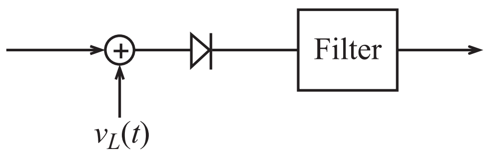

Figure 5.5: Diode switching mixer.

The local oscillator is a strong signal, which when positive causes the diode to conduct (switch closed), when negative causes the diode to be reverse biased (switch open). If the required turn-on voltage of the diode is v_d, then the diode is on when the switching signal is greater than v_d, as shown in Fig. 5.6.

For simplicity, we will assume that the switching voltage is strong enough that we can neglect the tiny turn-on voltage V_d, which means that when $v_L > 0$ the diode conducts. Mathematically, we can write

$$v_o(t) = [v_L(t) + v_i(t)]\, S_s(t) \tag{5.5}$$

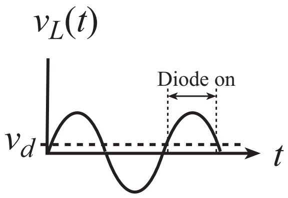

Figure 5.6: Diode voltage over one cycle of the switching signal.

where

$$S_s(t) = \begin{cases} 1 & v_L > 0 \\ 0 & v_L < 0 \end{cases}$$
$$= \frac{1}{2} + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin\left(\frac{n\pi}{2}\right)\cos(n\omega_L t) \tag{5.6}$$

Therefore, if we assume that $v_i(t) = V_i \cos \omega_i t$ and $v_L(t) = V_L \cos \omega_L t$, then

$$v_o(t) = \frac{1}{2}v_L(t) + \frac{1}{2}v_i(t) + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin\left(\frac{n\pi}{2}\right)\{V_L\cos(n\omega_L t)\cos(\omega_L t) + V_i\cos(n\omega_L t)\cos(\omega_i t)\}$$
$$= \frac{1}{2}v_L(t) + \frac{1}{2}v_i(t) + \frac{1}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin\left(\frac{n\pi}{2}\right)\{V_L\cos[(n-1)\omega_L t] + V_L\cos[(n+1)\omega_L t] +$$
$$V_i\cos[(n\omega_L - \omega_i)t] + V_i\cos[(n\omega_L + \omega_i)t]\} \tag{5.7}$$

Note that the $\sin(n\pi/2)$ term is zero for n even. Therefore, we will have signals at

1. ω_i
2. ω_L
3. $m\omega_L$ for m even
4. $n\omega_L \pm \omega_i$ for n odd

The desired term will be either $\omega_L + \omega_i$ for upconversion or $\omega_L - \omega_i$ for downconversion. The remaining undesired components will need to be filtered out.

5.2 Single Balanced Mixers

The simple mixer introduced above is a single-ended mixer. We have shown that this mixer produces a large variety of undesired signals. If we use a more balanced configuration, then some of these undesired signals can be suppressed. For example, consider the single-balanced mixer shown below:

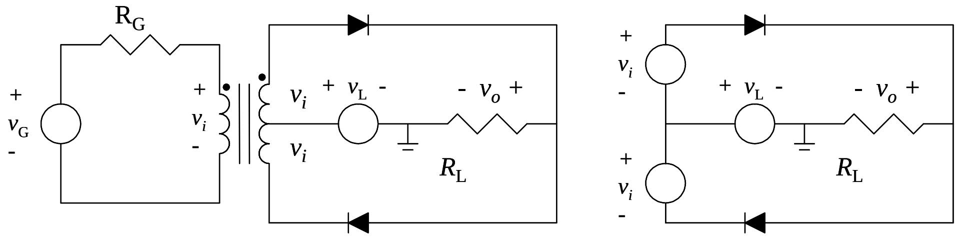

Figure 5.7: Single-balanced diode switching-type mixer.

For this circuit,

$$v_o = \begin{cases} v_L + v_i & v_L > 0 \\ v_L - v_i & v_L < 0 \end{cases}$$
$$= v_L + v_i S_b(t) \tag{5.8}$$

where

$$S_b(t) = \begin{cases} +1 & v_L > 0 \\ -1 & v_L < 0 \end{cases}$$
$$= \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin\left(\frac{n\pi}{2}\right)\cos(n\omega_L t) \tag{5.9}$$

So,

$$v_o(t) = v_L(t) + \frac{2V_i}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin\left(\frac{n\pi}{2}\right)\{\cos[(n\omega_L - \omega_i)t] + \cos[(n\omega_L + \omega_i)t]\} \tag{5.10}$$

We now have signals at

1. ω_L
2. $n\omega_L \pm \omega_i$ for n odd

The balanced configuration has removed many of our undesired components. This tends to be a good choice for downconversion, since $\omega_L >> \omega_L - \omega_i$, so it is easy to filter out the undesirable signals at ω_L and $n\omega_L \pm \omega_i$ for $n > 1$.

If we augment this single-balanced design as follows: we can ALSO suppress the LO. In this case,

$$v_o(t) = v_i(t)S_s(t) = \frac{1}{2}v_i(t) + \frac{V_i}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin\left(\frac{n\pi}{2}\right)\{\cos[(n\omega_L - \omega_i)t] + \cos[(n\omega_L + \omega_i)t]\} \tag{5.11}$$

This is a good choice for upconversion since $\omega_L >> \omega_i$.

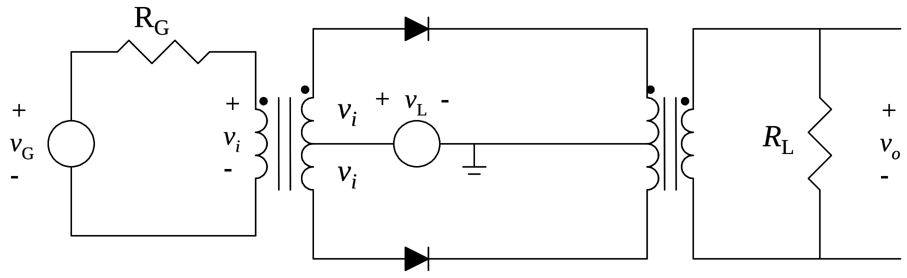

Figure 5.8: Alternate single-balanced diode switching-type mixer.

5.3 Double Balanced Mixer

To suppress both the signal and LO frequencies, we must go to a double balanced design:

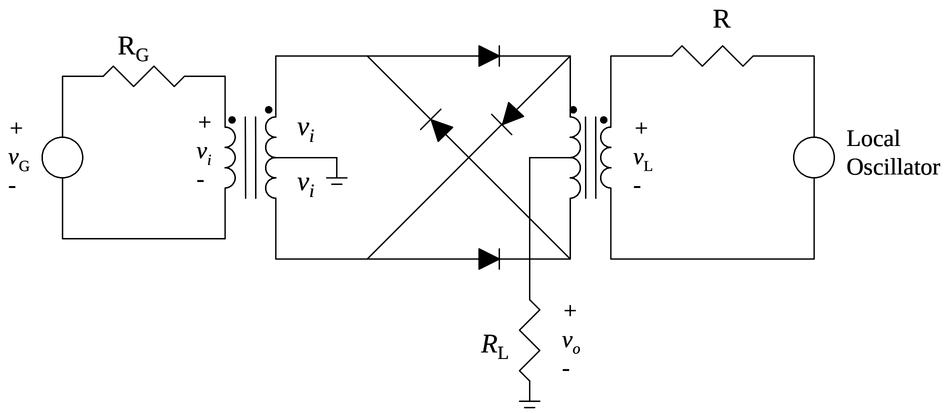

Figure 5.9: Double-balanced diode switching-type mixer.

For $v_L > 0$, we can simplify the circuit by removing the diodes that are off:

For the simplified circuit ($v_L > 0$):

$$v_i - (i_1 - i_2)R_L + v_L - r_d i_1 = 0 \tag{5.12}$$

$$v_i - (i_1 - i_2)R_L - v_L + r_d i_2 = 0 \tag{5.13}$$

Adding these equations leads to

$$2v_i - 2R_L(i_1 - i_2) - r_d(i_1 - i_2) = 0 \tag{5.14}$$

$$i_1 - i_2 = \frac{v_i}{R_L + r_d/2} = -\frac{v_o}{R_L} \tag{5.15}$$

For $v_L > 0$, we have

$$\frac{v_o}{v_i} = -\frac{R_L}{R_L + r_d/2} \tag{5.16}$$

A similar analysis for $v_L < 0$ leads to

$$\frac{v_o}{v_i} = \frac{R_L}{R_L + r_d/2} \tag{5.17}$$

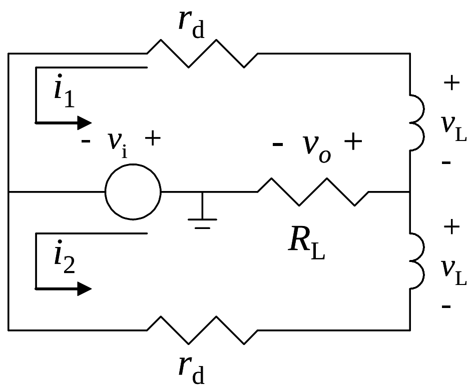

Figure 5.10: Simplified double-balanced mixer circuit for $v_L > 0$.

Combining these two results leads to

$$\begin{aligned} v_o &= \frac{R_L}{R_L + r_d/2} v_i S_b(t) \\ &= \frac{R_L}{R_L + r_d/2} \frac{2V_i}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \sin\left(\frac{n\pi}{2}\right) \{\cos[(n\omega_L - \omega_i)t] + \cos[(n\omega_L + \omega_i)t]\} \end{aligned} \quad (5.18)$$

This shows that the double balanced mixer eliminates both the LO and input signals at the output.

5.3.1 Conversion Loss

For the double-balanced mixer,

$$R_{in} = \frac{v_i}{i_1 - i_2} = R_L + r_d/2 \approx R_L \quad (5.19)$$

The maximum available power from the source is

$$P_i = \frac{V_p^2}{8R_L} \quad (5.20)$$

where V_p is the peak value of the source sinusoidal signal. The peak of the output voltage in a single sideband is then

$$V_o = \frac{2V_i}{\pi} = \frac{V_p}{\pi} \quad (5.21)$$

due to the voltage division of V_p. The output power is

$$P_o = \frac{V_p^2}{2\pi^2 R_L} \quad (5.22)$$

and the conversion loss is

$$L = \frac{P_i}{P_o} = \frac{\pi^2}{4} \quad (5.23)$$

$$L_{\text{dB}} = 10 \log\left(\frac{\pi^2}{4}\right) = 3.92 \text{ dB} \approx 4 \text{ dB} \quad (5.24)$$

For the single-balanced mixer,

$$L_{\text{dB}} = 9.94 \text{ dB} \approx 10 \text{ dB} \quad (5.25)$$