3.2 Unilateral Amplifier Design - Gain Circles

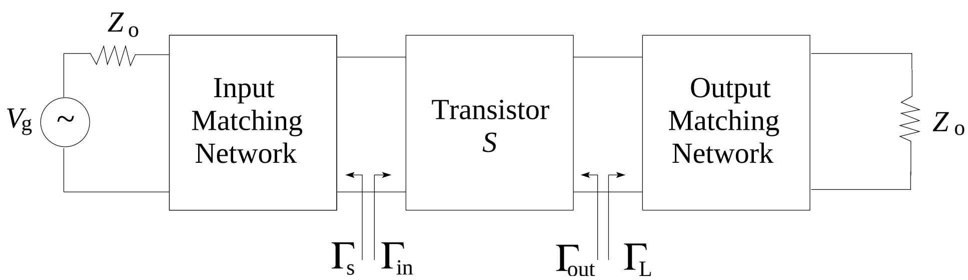

Figure 3.2: Amplifier with device and source and load matching networks.

When designing an amplifier, we typically design matching networks at the source and load as shown in Fig. (3.2) with values of Γ_s and Γ_L such that a given set of design criteria for the amplifier are met (gain, stability, noise performance, bandwidth, etc.).

In order to obtain a specified gain, we can use the method of constant gain circles, which are circles of values of Γ_s and Γ_L which give constant gain. For a unilateral device,

$$G_{TU} = G_s G_o G_L \tag{3.36}$$

where

$$G_s = \frac{1-|\Gamma_s|^2}{|1-\Gamma_s S_{11}|^2} \qquad G_L = \frac{1-|\Gamma_L|^2}{|1-\Gamma_L S_{22}|^2} \tag{3.37}$$

The maximum values of the source and load gain factors are

$$\begin{aligned} G_{s,\max} &= G_s|_{\Gamma_s=S_{11}^*} \quad \text{(conjugate match)} \\ &= \frac{1-|S_{11}|^2}{|1-|S_{11}|^2|^2} = \frac{1}{1-|S_{11}|^2} \end{aligned} \tag{3.38}$$

$$\begin{aligned} G_{L,\max} &= G_s|_{\Gamma_L=S_{22}^*} \quad \text{(conjugate match)} \\ &= \frac{1}{1-|S_{22}|^2} \end{aligned} \tag{3.39}$$

In terms of the maximum values, we define normalized gains,

$$g_s = \frac{G_s}{G_{s,\max}} = \frac{(1-|\Gamma_s|^2)(1-|S_{11}|^2)}{|1-S_{11}\Gamma_s|^2} \tag{3.40}$$

$$g_L = \frac{G_L}{G_{L,\max}} = \frac{(1-|\Gamma_L|^2)(1-|S_{22}|^2)}{|1-S_{22}\Gamma_L|^2} \tag{3.41}$$

so that $0 \le g_s \le 1$ and $0 \le g_L \le 1$.

If we want to design for a specified value of g_s, then rearranging the expression for normalized gain gives

$$\begin{aligned} g_s|1-S_{11}\Gamma_s|^2 &= (1-|\Gamma_s|^2)(1-|S_{11}|^2) \\ g_s(1-S_{11}\Gamma_s - S_{11}^*\Gamma_s^* + |S_{11}|^2|\Gamma_s|^2) &= 1-|S_{11}|^2-|\Gamma_s|^2+|S_{11}\Gamma_s|^2 \\ (g_s|S_{11}|^2+1-|S_{11}|^2)|\Gamma_s|^2 - g_s(S_{11}\Gamma_s+S_{11}^*\Gamma_s^*) &= 1-|S_{11}|^2-g_s \end{aligned} \tag{3.42}$$

This can be placed in the form

$$\Gamma_s\Gamma_s^* - \frac{g_s(S_{11}\Gamma_s + S_{11}^*\Gamma_s^*)}{1-(1-g_s)|S_{11}|^2} = \frac{1-|S_{11}|^2 - g_s}{1-(1-g_s)|S_{11}|^2} \tag{3.43}$$

We will now show that this is an equation for a circle in the Γ_s plane. The equation for a circle with center C and radius r in the complex plane is

$$\begin{aligned} r^2 &= |z-C|^2 \\ &= (z-C)(z^*-C^*) \\ &= zz^* - (C^*z + Cz^*) + |C|^2 \end{aligned} \tag{3.44}$$

By comparing Eqs. (3.43) and (3.44), we can see that Γ_s lies on a circle with center at

$$C_s = \frac{g_s S_{11}^*}{1-(1-g_s)|S_{11}|^2} \tag{3.45}$$

The radius of the circle is given by

$$\begin{aligned} r_s^2 &= |C_s|^2 + \frac{1-|S_{11}|^2 - g_s}{1-(1-g_s)|S_{11}|^2} \\ &= \frac{(1-|S_{11}|^2 - g_s)[1-(1-g_s)|S_{11}^2|] + g_s^2|S_{11}|^2}{[1-(1-g_s)|S_{11}|^2]^2} \\ &= \frac{(1-g_s)(|S_{11}|^4 - 2|S_{11}|^2 + 1)}{[1-(1-g_s)|S_{11}|^2]^2} \\ &= \frac{(1-g_s)(1-|S_{11}|^2)^2}{[1-(1-g_s)|S_{11}|^2]^2} \end{aligned} \tag{3.46}$$

so that

$$r_s = \frac{\sqrt{1-g_s}(1-|S_{11}|^2)}{1-(1-g_s)|S_{11}|^2} \tag{3.47}$$

The center C_L and radius r_L of the Γ_L circle for a constant value of g_L can be obtained from Eqs. (3.45) and (3.47) by replacing g_s with g_L and S_{11} with S_{22}.

These results lead to a design procedure for a desired value of the gain:

1. From $G_{TU} = G_s|S_{21}|^2 G_L$, determine desired values for G_s and G_L. One approach is to conjugate match the input port so that $G_s = G_{s,\max}$, and then choose G_L to obtain the desired gain.

2. Compute the normalized gains g_s and g_L.

3. Compute the load gain circle center and radius, C_L and r_L. If the source is not conjugate matched, compute the source gain circle center and radius C_s and r_s also.

4. Choose convenient values of Γ_s and Γ_L on these circles. Any value on the circle will meet the gain target, so we need some way to choose a particular value. One possibility is to choose the reflection coefficient with smallest magnitude ($|\Gamma|$ closest to zero). Later, we will consider other performance metrics such as noise figure that will dictate the choices of Γ_s and Γ_L.

5. Find the source network impedance and load network impedance using

$$Z_s = Z_o \frac{1+\Gamma_s}{1-\Gamma_s} \tag{3.48}$$

$$Z_L = Z_o \frac{1+\Gamma_L}{1-\Gamma_L} \tag{3.49}$$

   If $g_s = 1$, then $C_s = S_{11}^*$ and $r_s = 0$, so that $\Gamma_s = S_{11}^*$, which is what we expect, since this is a conjugate match.

   If we conjugate match both the input and output, then we can have both $G_s > 1$ and $G_L > 1$. How is this possible with passive matching networks?

3.3 Stability

After gain, the next key amplifier concept is stability. If an amplifier is unstable, noise feedback will lead to oscillation. Stability can be determined from the reflection coefficients $\Gamma_{\rm in}$ and $\Gamma_{\rm out}$ looking into the input and output of the transistor. If the magnitude of one or both of these reflection coefficients is greater than unity, then the amplifier is unstable.

Since $\Gamma_{\rm in}$ and $\Gamma_{\rm out}$ depend on the reflection coefficients Γ_s and Γ_L looking from the device into the source and load, the matching networks determine the stability of the amplifier. There are two possible situations:

1. Unconditional stability: $|\Gamma_{\rm in}| < 1$ and $|\Gamma_{\rm out}| < 1$ for all passive source and load impedances ($|\Gamma_s| < 1$, $|\Gamma_L| < 1$).
2. Conditional stability: $|\Gamma_{\rm in}| < 1$ and $|\Gamma_{\rm out}| < 1$ only for a certain range of source and load impedances. This is also called potentially unstable. For this case, we design the source and load matching networks to be such that the amplifier is in the stable region.

In order to determine the stability of an amplifier, we need to examine the reflection coefficients looking into the two device ports. The stability conditions are

$$|\Gamma_{\rm in}| = \left|S_{11} + \frac{S_{12}\Gamma_L S_{21}}{1 - S_{22}\Gamma_L}\right| < 1 \tag{3.50}$$

$$|\Gamma_{\rm out}| = \left|S_{22} + \frac{S_{12}\Gamma_s S_{21}}{1 - S_{11}\Gamma_s}\right| < 1 \tag{3.51}$$

We will use these to find out what values Γ_s and Γ_L may take on in order to have stability.

Unilateral device. For a unilateral device, $S_{12} = 0$, and the stability conditions become

$$|S_{11}| < 1 \tag{3.52}$$

$$|S_{22}| < 1 \tag{3.53}$$

which are a function of the device only, and not the source and load matching networks.

Bilateral device. The bilateral case is more complicated, because stability depends on the source and load matching networks. The boundary of the region of stability is defined by

$$|\Gamma_{\rm in}| = |\Gamma_{\rm out}| = 1 \tag{3.54}$$

The first condition becomes

$$|S_{11}(1 - S_{22}\Gamma_L) + S_{12}\Gamma_L S_{21}| = |1 - S_{22}\Gamma_L| \tag{3.55}$$

$$|S_{11} - \Delta\Gamma_L| = |1 - S_{22}\Gamma_L| \tag{3.56}$$

where $\Delta = S_{11}S_{22} - S_{12}S_{21}$ is the determinant of the S-parameter matrix of the device. Squaring both sides of the last expression leads to

$$|S_{11}|^2 + |\Delta|^2|\Gamma_L|^2 - \Delta\Gamma_L S_{11}^* + \Delta^*\Gamma_L^* S_{11} = 1 + |S_{22}|^2|\Gamma_L|^2 - (S_{22}^*\Gamma_L^* + S_{22}\Gamma_L) \tag{3.57}$$

Combining terms containing Γ_L gives

$$|\Gamma_L|^2 - \frac{(S_{22} - \Delta S_{11}^*)\Gamma_L + (S_{22}^* - \Delta^* S_{11})\Gamma_L^*}{|S_{22}|^2 - |\Delta|^2} = \frac{|S_{11}|^2 - 1}{|S_{22}|^2 - |\Delta|^2} \tag{3.58}$$

If we compare this to Eq. (3.44) for a circle in the complex plane, we find that the center and radius of the circle in the Γ_L plane is

$$C_L = \frac{S_{22}^* - \Delta^* S_{11}}{|S_{22}|^2 - |\Delta|^2} \tag{3.59}$$

$$r_L = \left| \frac{S_{12}S_{21}}{|S_{22}|^2 - |\Delta|^2} \right| \tag{3.60}$$

For the condition on Γ_{out}, we get similar expressions for the center C_s and radius r_s of the stability circle in the Γ_s plane, with S_{11} and S_{22} interchanged. Why does the input stability condition lead to a circle for Γ_L, which is on the output side, and the output stability condition lead to a condition on Γ_s, the reflection coefficient looking into the source network?

If we have a matched load, then $Z_L = Z_0$ and $\Gamma_L = 0$, so that

$$|\Gamma_{\text{in}}| = |S_{11}| \tag{3.61}$$

If $|S_{11}| < 1$, the center of the Smith chart represents a stable value of Γ_L. Otherwise, the center of the Smith chart is in the unstable region. This can be used to determine whether the inside or the outside of a stability circle represents the stable region for a device.

In practice, it is good to be well inside the stable region, and to be sure that Γ_s and Γ_L are inside the stable region over a range of frequencies near the design frequency.

Unconditional stability. If a device is unconditionally stable, then the entire Smith chart ($|\Gamma| < 1$) is inside the stability circles. It can be shown that a device is unconditionally stable if

$$\frac{1 - |S_{11}|^2}{|S_{22} - S_{11}^*\Delta| + |S_{12}S_{21}|} > 1 \tag{3.62}$$

and the greater this quantity is, the more stable the device.

3.4 Bilateral Design

If $S_{12} \neq 0$, the constant gain circle design approach used above needs some changes. For a bilateral device, the transducer gain cannot be separated into independent factors for the source and load ports, so to achieve a given value of the gain we would have to adjust both Γ_s and Γ_L at the same time. To avoid this difficulty, we will work with either the power gain G_P or the available power gain G_A since they are independent of the source or load, respectively.

The power gain is

$$G_P = \frac{P_L}{P_{\rm in}} = \frac{1}{1-|\Gamma_{\rm in}|^2}|S_{21}|^2\frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2} \tag{3.63}$$

The goal here is a way to find Γ_L given a specified power gain. First, we need to write $\Gamma_{\rm in}$ in terms of Γ_L, using

$$|\Gamma_{\rm in}| = \left|S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1-S_{22}\Gamma_L}\right| = \left|\frac{S_{11} - S_{11}S_{22}\Gamma_L + S_{12}S_{21}\Gamma_L}{1-S_{22}\Gamma_L}\right| = \left|\frac{S_{11}-\Delta\Gamma_L}{1-S_{22}\Gamma_L}\right| \tag{3.64}$$

With this result, the power gain can be expressed as

$$G_P = \frac{1}{1-\left|\frac{S_{11}-\Delta\Gamma_L}{1-S_{22}\Gamma_L}\right|^2}|S_{21}|^2\frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2} \tag{3.65}$$

which is a function of Γ_L and the device parameters only. We will use this expression to design the value of Γ_L to achieve a specified power gain, and then use a conjugate match for the source network.

The power gain normalized by the intrinsic transistor gain $|S_{21}|^2$ is

$$\begin{aligned} g_P &= \frac{G_P}{|S_{21}|^2} \\ &= \frac{1-|\Gamma_L|^2}{1-S_{22}\Gamma_L - S_{22}^*\Gamma_L^* + |S_{22}\Gamma_L|^2 - |S_{11}|^2 + S_{11}^*\Delta\Gamma_L + S_{11}\Delta^*\Gamma_L^* - |\Delta\Gamma_L|^2} && (3.66) \\ &= \frac{1-|\Gamma_L|^2}{1-|S_{11}|^2 + |\Gamma_L|^2(|S_{22}|^2-|\Delta|^2) - \Gamma_L(S_{22}-\Delta S_{11}^*) - \Gamma_L^*(S_{22}^*-\Delta^* S_{11})} && (3.67) \end{aligned}$$

Cross multiplying and rearranging leads to

$$|\Gamma_L|^2 - \Gamma_L\frac{g_P(S_{22}-\Delta S_{11}^*)}{D} - \Gamma_L^*\frac{g_P(S_{22}^*-\Delta^* S_{11})}{D} + \frac{g_P(1-|S_{11}|^2)}{D} = \frac{1}{D} \tag{3.68}$$

where $D = 1 + g_P(|S_{22}|^2 - |\Delta|^2)$. We recognize this as a circle in the Γ_L plane. The center and radius are

$$\begin{aligned} C_P &= \frac{g_P(S_{22}^*-\Delta^* S_{11})}{1+g_P(|S_{22}|^2-|\Delta|^2)} && (3.69) \\ r_P &= \frac{[1-2k|S_{12}S_{21}|g_P + |S_{12}S_{21}|^2 g_P^2]^{1/2}}{|1+g_P(|S_{22}|^2-|\Delta|^2)|} && (3.70) \\ k &= \frac{1-|S_{11}|^2-|S_{22}|^2+|\Delta|^2}{2|S_{12}S_{21}|} && (3.71) \end{aligned}$$

This is similar to the unilateral design procedure we developed earlier, except that we do not know the range of possible values for g_P.

In order to understand the range of attainable gains in the bilateral case, we need to find out the maximum value of g_P. Let the quantity inside the square brackets in the expression for r_P be denoted by $f(g_P)$, so that

$$f(g_P) = 1 - 2k|S_{12}S_{21}|g_P + |S_{12}S_{21}|^2 g_P^2 \tag{3.72}$$

Since we must have $r_P > 0$, then $f(g_P) > 0$ as well. We can also see that $f(0) = 1$. Using the quadratic formula, the zeros of $f(g_P)$ are

$$g_{1,2} = \frac{1}{|S_{12}S_{21}|}[k \mp \sqrt{k^2 - 1}] \tag{3.73}$$

where g_1 corresponds to the upper sign and g_2 to the lower. Using the zeros, we can factor the polynomial into the product form

$$f(g_P) = |S_{12}S_{21}|^2 \left[g_P - \frac{1}{|S_{12}S_{21}|}(k - \sqrt{k^2 - 1})\right] \left[g_P - \frac{1}{|S_{12}S_{21}|}(k + \sqrt{k^2 - 1})\right] \tag{3.74}$$

$$= |S_{12}S_{21}|^2 (g_P - g_1)(g_P - g_2) \tag{3.75}$$

If we assume that $k > 1$ (which is true for a transistor that is unconditionally stable), then both zeros are positive.

Putting all of this together, we can see that $f(g_P)$ is a parabola that is one at $g_P = 0$, goes negative at g_1, and becomes positive again at g_2. Since $f(g_P)$ must be positive, we can see that the interval for physically meaningful values of the normalized gain is $0 \leq g_P \leq g_1$. We therefore have

$$g_{P,\max} = \frac{1}{|S_{12}S_{21}|}[k - \sqrt{k^2 - 1}] \tag{3.76}$$

$$G_{P,\max} = \left|\frac{S_{21}}{S_{12}}\right| [k - \sqrt{k^2 - 1}] \tag{3.77}$$

Also, since $r_P = 0$ at $g_P = g_{P,\max}$, C_P becomes

$$\Gamma_{ML} = \frac{g_{P,\max}(S_{22}^* - \Delta^* S_{11})}{1 + g_{P,\max}(|S_{22}|^2 - |\Delta|^2)} \tag{3.78}$$

which is the value of Γ_L that maximizes G_P. These results provide a design approach for a bilateral transistor amplifier.

Design Procedure

1. For the desired power gain G_P, compute the normalized gain g_p and plot the resulting gain circle on the Γ_L plane.

2. *Load reflection coefficient:* Choose a value of Γ_L on the gain circle.

3. *Source reflection coefficient:* Compute Γ_{in} using the selected value for Γ_L. Conjugate match the source, so that $\Gamma_S = \Gamma_{\text{in}}^*$. For this matching condition, $P_{\text{in}} = P_{\text{avs}}$ and therefore $G_T = G_P$.

Our choice of Γ_L produces a value for Γ_{in} which in turn determines Γ_S. This value of Γ_S results in a value of Γ_{out} which determines the output VSWR. If we don't like the output VSWR that we obtain for some reason, we can always choose a different value of Γ_L.

We won't take the time to prove this, but it can be shown that if we pick $\Gamma_L = \Gamma_{ML}$, then $\Gamma_{\text{out}}^* = \Gamma_L$. In other words, maximizing the gain is equivalent to conjugate matching the input and output.

3.5 Noise in Communications Systems

There are several types of noise that are included in communication systems.

1. Thermal Noise (Johnson or Nyquist noise): Created by thermal vibration of bound charges.
2. Shot Noise: Random fluctuations of charge carriers in a solid-state device.
3. Flicker Noise ($1/f$ noise): Occurs in solid-state components. The noise power varies as $1/f$.
4. Plasma Noise: Random motion of charges in an ionized gas.

Thermal noise tends to be dominant in most systems, so we will concentrate on this.

Consider a resistor with resistance R at a temperature T (in Kelvin). The kinetic energy of the electrons is proportional to T. The random motion of the electrons create voltage fluctuations at the resistor terminals. The voltage has zero average, but the RMS value is given by Planck's blackbody radiation equation

$$\overline{v}_n = \sqrt{\frac{4hfBR}{e^{hf/k_BT} - 1}} \tag{3.79}$$

where

B Bandwidth in Hertz
h Planck's constant = 6.546×10^{-34} J·sec
k_B Boltzmann's constant = 1.380×10^{-23} J/K
f frequency (Hz)

If the frequency is large, say $f = 100$ GHz, and the temperature is low, so that $T = 100$K, then

$$hf = 6.5 \times 10^{-23} \ll k_BT = 1.38 \times 10^{-21} \tag{3.80}$$

This means that the exponent hf/k_BT is very small. The inequality gets even larger for microwave frequencies at room temperature (T = 273 K). Because of this, at microwave frequencies the exponential can be approximated by the first two terms of the Taylor series,

$$e^{hf/k_BT} \approx 1 + \frac{hf}{k_BT} \tag{3.81}$$

This simplifies the RMS voltage to

$$\overline{v}_n \approx \sqrt{\frac{4hfBR}{1 + hf/k_BT - 1}} = \sqrt{4k_BTBR} \tag{3.82}$$

In this approximation, $\overline{v}_n$ is independent of frequency. For this reason, the thermal noise signal is called "white noise". We generally model the noise voltage as a random variable with a zero mean Gaussian distribution and variance $\overline{v}_n^2$. Given multiple noise sources, the distributions are independent. Mathematically, this means that if you combine multiple noise sources, the variance of the sum is equal to the sum of the variances (we add the noise powers, not the voltages).