Chapter 3

Amplifiers

For low frequency amplifier design, a transistor can be modeled using an equivalent circuit. At microwave frequencies, reflections from the input and output ports become important, so a network description characterized by S-parameters as a function of frequency is required. The key parameters of a transistor for microwave amplification are

- Gain as a function of frequency
- $f_T =$ frequency at which gain drops to unity
- S-parameters as a function of frequency
- Noise figure: characterizes noise added to signal by the device ($\mathrm{SNR}_{\mathrm{out}} < \mathrm{SNR}_{\mathrm{in}}$).

f_T determines the usable bandwidth of the transistor, the S-parameters are used to design matching networks to match to the transistor and determine the gain and stability of the amplifier, and the noise figure is used to determine the amount of noise produced by the transistor.

3.1 Gain

The most important quantity for a microwave amplifier is power gain. In order to analyze amplifier gain, we first need to determine the gain in terms of the S-parameters of the transistor. Because there are forward and reverse waves at the input and output ports, unlike in circuit theory, there several different power measures that can be used to define gain:

- $P_{\mathrm{in}} =$ power delivered to network
- $P_{\mathrm{avs}} =$power available from source
- $P_L =$ power delivered to load
- $P_{\mathrm{avn}} =$ power available from network

Available power is the maximum power that can be supplied with over all possible load impedances. The actual power delivered may be smaller than the available power due to reflections and is less than or equal to the available power.

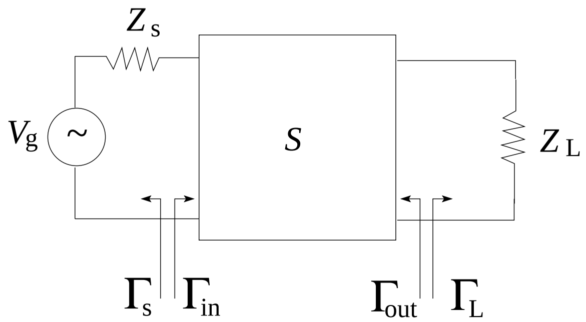

Figure 3.1: A microwave amplifier connected to a source at the input (port 1) and a load impedance at the output (port 2).

We first need to find the reflection coefficients looking into ports 1 and 2, as shown in Fig. 3.1. The source and load reflection coefficients are

$$\Gamma_s = \frac{Z_s - Z_0}{Z_s + Z_0} \qquad \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} \tag{3.1}$$

Given these definitions, we can write the amplitudes b_1 and b_2 of the waves exiting ports 1 and 2 as

$$b_1 = S_{11}a_1 + S_{12}a_2 = S_{11}a_1 + S_{12}\Gamma_L b_2 \tag{3.2}$$

$$b_2 = S_{21}a_1 + S_{22}a_2 = S_{21}a_1 + S_{22}\underbrace{\Gamma_L b_2}_{a_2} \tag{3.3}$$

Solving the second equation for b_2 gives

$$b_2 = \frac{S_{21}}{1 - S_{22}\Gamma_L}a_1 \tag{3.4}$$

Using this expression in the first equation gives

$$b_1 = S_{11}a_1 + S_{12}\Gamma_L \frac{S_{21}}{1 - S_{22}\Gamma_L}a_1 \tag{3.5}$$

The ratio b_1/a_1 gives the reflection coefficient looking into the input port:

$$\Gamma_{\text{in}} = \frac{b_1}{a_1} = S_{11} + \frac{S_{12}\Gamma_L S_{21}}{1 - S_{22}\Gamma_L} = \frac{Z_{\text{in}} - Z_0}{Z_{\text{in}} + Z_0} \tag{3.6}$$

Repeating this procedure for a source at port 2, we find that

$$\Gamma_{\text{out}} = S_{22} + \frac{S_{12}\Gamma_s S_{21}}{1 - S_{11}\Gamma_s} \tag{3.7}$$

This expression can be understood in an intuitive way. The first term accounts for reflections at port 2, and would be the entire reflection coefficient if port 1 were matched. In the second term, S_{12} takes the signal from port 2 to port 1, Γ_s reflects the signal from the source impedance, S_{21} returns the signal to port 2, and the denominator takes into account multiple reflections between the ports.

We can compute the various power quantities in terms of these reflection coefficients, using

$$\begin{aligned} P_{\text{in}} &= |a_1|^2/2 - |b_1|^2/2 \\ &= |a_1|^2/2\left(1 - |\Gamma_{in}|^2\right) \end{aligned} \tag{3.8}$$

$$\begin{aligned} P_L &= |b_2|^2/2 - |a_2|^2/2 \\ &= |b_2|^2/2\left(1 - |\Gamma_L|^2\right) \end{aligned} \tag{3.9}$$

Now, we need to find a_1 and b_2. We can get the voltage at port 1 using a voltage divider,

$$V_1 = V_s \frac{Z_{\text{in}}}{Z_s + Z_{\text{in}}} \tag{3.10}$$

$$= \sqrt{Z_0}(a_1 + b_1) \quad \text{by the definition of generalized S-parameters} \tag{3.11}$$

$$= \sqrt{Z_0}a_1(1 + \Gamma_{\text{in}}) \tag{3.12}$$

Substituting the definition of Z_{in} in terms of Γ_{in} into this expression gives

$$V_s \frac{(1+\Gamma_{\text{in}})Z_0}{Z_s(1-\Gamma_{\text{in}}) + Z_0(1+\Gamma_{\text{in}})} = \sqrt{Z_0}a_1(1+\Gamma_{\text{in}}) \tag{3.13}$$

Solving for a_1 and putting the expression in terms of reflection coefficients gives

$$\begin{aligned} a_1 &= \frac{Z_0}{Z_s(1-\Gamma_{\text{in}}) + Z_0(1+\Gamma_{\text{in}})} \frac{V_s}{\sqrt{Z_0}} \\ &= \frac{Z_0}{Z_0\frac{1+\Gamma_s}{1-\Gamma_s}(1-\Gamma_{\text{in}}) + Z_0(1+\Gamma_{\text{in}})} \frac{V_s}{\sqrt{Z_0}} \\ &= \frac{1-\Gamma_s}{(1+\Gamma_s)(1-\Gamma_{\text{in}}) + (1-\Gamma_s)(1+\Gamma_{\text{in}})} \frac{V_s}{\sqrt{Z_0}} \\ &= \frac{1}{2}\frac{1-\Gamma_s}{1-\Gamma_{\text{in}}\Gamma_s}\frac{V_s}{\sqrt{Z_0}} \end{aligned} \tag{3.14}$$

If we put this into the expression for P_{in}, we find that

$$\begin{aligned} P_{\text{in}} &= \frac{1}{2}\left|\frac{1}{2}\frac{1-\Gamma_s}{1-\Gamma_{\text{in}}\Gamma_s}\frac{V_s}{\sqrt{Z_0}}\right|^2(1-|\Gamma_{\text{in}}|^2) \\ &= \frac{|V_s|^2}{8Z_0}\frac{|1-\Gamma_s|^2}{|1-\Gamma_{\text{in}}\Gamma_s|^2}(1-|\Gamma_{\text{in}}|^2) \end{aligned} \tag{3.15}$$

Using (3.4) together with (3.14) gives the amplitude of the wave propagating out of port 2,

$$b_2 = \frac{S_{21}}{1-S_{22}\Gamma_L}a_1 = \frac{V_s}{2\sqrt{Z_0}}\frac{S_{21}(1-\Gamma_s)}{(1-\Gamma_{\text{in}}\Gamma_s)(1-S_{22}\Gamma_L)} \tag{3.16}$$

This can be used to obtain the power delivered to the load,

$$P_L = \frac{|V_s|^2}{8Z_0}|S_{21}|^2\frac{(1-|\Gamma_L|^2)|1-\Gamma_s|^2}{|1-S_{22}\Gamma_L|^2|1-\Gamma_s\Gamma_{\text{in}}|^2} \tag{3.17}$$

Power Gain

Using these results, the power gain of the amplifier is

$$\begin{aligned} G_P = \frac{P_L}{P_{\text{in}}} &= |S_{21}|^2 \frac{(1-|\Gamma_L|^2)|1-\Gamma_s|^2}{|1-S_{22}\Gamma_L|^2|1-\Gamma_s\Gamma_{\text{in}}|^2} \frac{|1-\Gamma_s\Gamma_{\text{in}}|^2}{|1-\Gamma_s|^2(1-|\Gamma_{\text{in}}|^2)} \\ &= |S_{21}|^2 \frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2(1-|\Gamma_{\text{in}}|^2)} \end{aligned} \tag{3.18}$$

This is the gain of the amplifier in terms of power delivered to the load relative to power coming into the amplifier input port.

In this expression, if the load reflection coefficient is one, then the gain vanishes, as expected. If the load reflection coefficient is zero, then

$$G_P = |S_{21}|^2 \frac{1}{1-|\Gamma_{\text{in}}|^2} = |S_{21}|^2 \frac{1}{1-|S_{11}|^2} \tag{3.19}$$

Note, however, that $\Gamma_L = 0$ does not mean the power gain is maximized, since it simply means $Z_L = Z_0$ whereas maximum power gain occurs for a conjugate match condition (we will see this more clearly later). More generally, the power gain is equal to $|S_{21}|^2$ scaled by a factor which takes into account reflections at the load (which means G_P can be larger than $|S_{21}|^2$).

Transducer Power Gain

The problem with power gain as defined in Eq. (3.19) is that it does not consider the match between the source and the input impedance to the network. In some cases, therefore, a more meaningful measure of gain is the power dissipated by the load relative to the maximum power that the source can supply. This is the transducer power gain,

$$G_T = \frac{P_L}{P_{\text{avs}}} \tag{3.20}$$

If we consider a source with impedance Z_s driving a line with input impedance Z_{in}, the maximum power transfer occurs when Z_{in} is the complex conjugate of Z_s, so that

$$Z_{\text{in}} = Z_s^* \tag{3.21}$$

This can be proved by taking the derivative of the power delivered to the line with respect to the real and imaginary parts of Z_{in} and setting the derivatives to zero. The imaginary parts of Z_{in} and Z_s are equal in magnitude and opposite in sign, which is what happens with the impedances of the inductor and capacitor in an LCR circuit at resonance. This is called a **conjugate match**.

With a conjugate match to the source impedance, the power available from the source is

$$\begin{aligned} P_{\text{avs}} &= P_{\text{in}}|_{\Gamma_{\text{in}}=\Gamma_s^*} \quad \text{(conjugate match)} \\ &= \frac{|V_s|^2}{8Z_0} \frac{|1-\Gamma_s|^2}{|1-\underbrace{\Gamma_s^*}_{\text{cm}}\Gamma_s|^2}(1-|\Gamma_s|^2) \\ &= \frac{|V_s|^2}{8Z_0} \frac{|1-\Gamma_s|^2}{1-|\Gamma_s|^2} \end{aligned} \tag{3.22}$$

The transducer gain is then

$$\begin{aligned} G_T &= |S_{21}|^2 \frac{(1-|\Gamma_L|^2)|1-\Gamma_s|^2}{|1-S_{22}\Gamma_L|^2|1-\Gamma_s\Gamma_{\text{in}}|^2} \frac{1-|\Gamma_s|^2}{|1-\Gamma_s|^2} \\ &= |S_{21}|^2 \frac{(1-|\Gamma_s|^2)(1-|\Gamma_L|^2)}{|1-S_{22}\Gamma_L|^2|1-\Gamma_s \underbrace{\Gamma_{\text{in}}}_{\neq \text{cm}}|^2} \end{aligned} \tag{3.23}$$

Notice that the input reflection coefficient in the power delivered to the load does not assume a conjugate match at the input. This is because transducer gain does not mean that we have actually conjugate matched to the input impedance. Instead, we just want to compute the gain relative to the input power we would have if the source were conjugate matched.

Available Power Gain

Another difficulty with power gain in Eq. (3.19) is that if the load is not well matched to the network output impedance, then the power gain is small, even though the amplifier is capable of delivering more power to a better matched load. If we want to characterize the amplifier independently of the load impedance, we can define a measure of gain in terms of the power delivered to a conjugate matched load. This is the available power gain,

$$G_A = \frac{P_{\text{avn}}}{P_{\text{avs}}} \tag{3.24}$$

which is the gain if both the source and load were conjugate matched. The power available from the amplifier network at port 2 is

$$\begin{aligned} P_{\text{avn}} &= P_L|_{\Gamma_L=\Gamma_{\text{out}}^*} \\ &= \frac{|V_s|^2}{8Z_0}|S_{21}|^2 \frac{(1-|\Gamma_{\text{out}}|^2)|1-\Gamma_s|^2}{|1-S_{22}\Gamma_{\text{out}}^*|^2|1-\Gamma_s\Gamma_{\text{in}}|^2} \end{aligned} \tag{3.25}$$

The available gain is

$$G_A = |S_{21}|^2 \frac{(1-|\Gamma_{\text{out}}|^2)(1-|\Gamma_s|^2)}{|1-S_{22}\Gamma_{\text{out}}^*|^2|1-\Gamma_s\Gamma_{\text{in}}|^2} \tag{3.26}$$

In order to simplify the expression for available power gain by eliminating Γ_{in}, we use

$$\begin{aligned} 1-\Gamma_s\Gamma_{\text{in}} &= 1-\Gamma_s\left(S_{11}+\frac{S_{12}\Gamma_L S_{21}}{1-S_{22}\Gamma_L}\right) \\ &= \frac{1-S_{22}\Gamma_L - S_{11}\Gamma_s + S_{11}S_{22}\Gamma_s\Gamma_L - S_{12}S_{21}\Gamma_s\Gamma_L}{1-S_{22}\Gamma_L} \\ &= \frac{1-S_{11}\Gamma_s}{1-S_{22}\Gamma_L}\left[1-\Gamma_L \underbrace{\left(S_{22}+\frac{S_{12}\Gamma_s S_{21}}{1-S_{11}\Gamma_s}\right)}_{\Gamma_{\text{out}}}\right] \\ &= \frac{1-S_{11}\Gamma_s}{1-S_{22}\Gamma_L}(1-\Gamma_L\Gamma_{\text{out}}) \end{aligned} \tag{3.27}$$

With this result, the available gain becomes

$$G_A = |S_{21}|^2 \frac{1-|\Gamma_s|^2}{|1-S_{11}\Gamma_s|^2(1-|\Gamma_{\text{out}}|^2)} \tag{3.28}$$

Special Cases

1. Both source and load impedances are equal to the Z_0 ($\Gamma_s = \Gamma_L = 0$). In this case,

$$G_T = |S_{21}|^2 \tag{3.29}$$

   It is important to be aware that if we instead choose a conjugate match at the source and load ($\Gamma_s = \Gamma_{\text{in}}^*$, $\Gamma_L = \Gamma_{\text{out}}^*$), the gain can be larger than $|S_{21}|^2$.

2. Unilateral device ($S_{12} = 0$ or very small). In this case, $\Gamma_{\text{in}} = S_{11}$ and $\Gamma_{\text{out}} = S_{22}$, and the transducer gain becomes

$$\begin{aligned} G_{TU} &= |S_{21}|^2 \frac{(1-|\Gamma_s|^2)(1-|\Gamma_L|^2)}{|1-S_{22}\Gamma_L|^2|1-\Gamma_s S_{11}|^2} \\ &= \underbrace{\frac{1-|\Gamma_s|^2}{|1-\Gamma_s S_{11}|^2}}_{\substack{\text{source} \\ G_s}} \underbrace{|S_{21}|^2}_{\substack{\text{device} \\ G_o}} \underbrace{\frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}}_{\substack{\text{load} \\ G_L}} \end{aligned} \tag{3.30}$$

   which can be broken up into a product of source, device, and load gain factors. Again, it is possible for G_s and G_L to be greater than one, depending on the source and load matches.

   Using the expression for transducer gain for a bilateral device, it can be shown that the error made in assuming that a transistor is unilateral is bounded by

$$\frac{1}{(1+U)^2} \le \frac{G_T}{G_{TU}} < \frac{1}{(1-U)^2} \tag{3.31}$$

   where

$$U = \frac{|S_{12}S_{21}S_{11}S_{22}|}{(1-|S_{11}|^2)(1-|S_{22}|^2)} \tag{3.32}$$

   is the unilateral figure of merit. The smaller U, the closer the device is to unilateral.

Summary

For a two-port amplifier, we define three gain quantities:

$$G_P = \frac{P_L}{P_{\text{in}}} \quad \text{(Power Gain)} \tag{3.33}$$

$$G_T = \frac{P_L}{P_{\text{avs}}} \quad \text{(Transducer Gain)} \tag{3.34}$$

$$G_A = \frac{P_{\text{avn}}}{P_{\text{avs}}} \quad \text{(Available Power Gain)} \tag{3.35}$$

In developing amplifier design procedures, we will use whichever type of gain is most convenient for a given problem. For unilateral amplifier design ($S_{12} = 0$), we will use transducer gain, and for bilateral design ($S_{12} \neq 0$) we will use power gain.

3.2 Unilateral Amplifier Design - Gain Circles

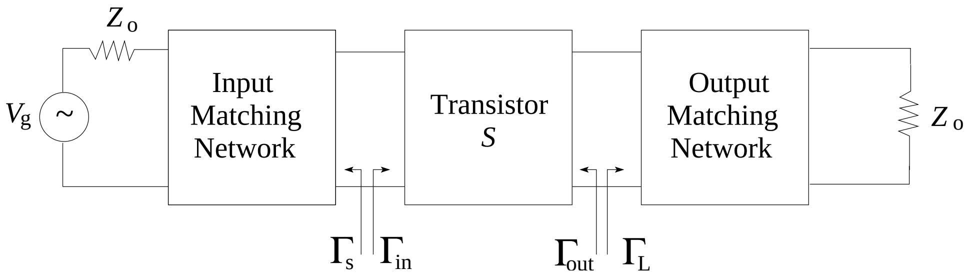

Figure 3.2: Amplifier with device and source and load matching networks.

When designing an amplifier, we typically design matching networks at the source and load as shown in Fig. (3.2) with values of Γ_s and Γ_L such that a given set of design criteria for the amplifier are met (gain, stability, noise performance, bandwidth, etc.).

In order to obtain a specified gain, we can use the method of constant gain circles, which are circles of values of Γ_s and Γ_L which give constant gain. For a unilateral device,

$$G_{TU} = G_s G_o G_L \tag{3.36}$$

where

$$G_s = \frac{1-|\Gamma_s|^2}{|1-\Gamma_s S_{11}|^2} \qquad G_L = \frac{1-|\Gamma_L|^2}{|1-\Gamma_L S_{22}|^2} \tag{3.37}$$

The maximum values of the source and load gain factors are

$$\begin{aligned} G_{s,\max} &= G_s|_{\Gamma_s=S_{11}^*} \quad \text{(conjugate match)} \\ &= \frac{1-|S_{11}|^2}{|1-|S_{11}|^2|^2} = \frac{1}{1-|S_{11}|^2} && (3.38) \\ G_{L,\max} &= G_s|_{\Gamma_L=S_{22}^*} \quad \text{(conjugate match)} \\ &= \frac{1}{1-|S_{22}|^2} && (3.39) \end{aligned}$$

In terms of the maximum values, we define normalized gains,

$$\begin{aligned} g_s &= \frac{G_s}{G_{s,\max}} = \frac{(1-|\Gamma_s|^2)(1-|S_{11}|^2)}{|1-S_{11}\Gamma_s|^2} && (3.40) \\ g_L &= \frac{G_L}{G_{L,\max}} = \frac{(1-|\Gamma_L|^2)(1-|S_{22}|^2)}{|1-S_{22}\Gamma_L|^2} && (3.41) \end{aligned}$$

so that $0 \le g_s \le 1$ and $0 \le g_L \le 1$.

If we want to design for a specified value of g_s, then rearranging the expression for normalized gain gives

$$\begin{aligned} g_s|1-S_{11}\Gamma_s|^2 &= (1-|\Gamma_s|^2)(1-|S_{11}|^2) \\ g_s(1-S_{11}\Gamma_s - S_{11}^*\Gamma_s^* + |S_{11}|^2|\Gamma_s|^2) &= 1-|S_{11}|^2 - |\Gamma_s|^2 + |S_{11}\Gamma_s|^2 \\ (g_s|S_{11}|^2 + 1 - |S_{11}|^2)|\Gamma_s|^2 - g_s(S_{11}\Gamma_s + S_{11}^*\Gamma_s^*) &= 1-|S_{11}|^2 - g_s && (3.42) \end{aligned}$$

This can be placed in the form

$$\Gamma_s\Gamma_s^* - \frac{g_s(S_{11}\Gamma_s + S_{11}^*\Gamma_s^*)}{1-(1-g_s)|S_{11}|^2} = \frac{1-|S_{11}|^2 - g_s}{1-(1-g_s)|S_{11}|^2} \tag{3.43}$$

We will now show that this is an equation for a circle in the Γ_s plane. The equation for a circle with center C and radius r in the complex plane is

$$\begin{aligned} r^2 &= |z-C|^2 \\ &= (z-C)(z^*-C^*) \\ &= zz^* - (C^*z + Cz^*) + |C|^2 \end{aligned} \tag{3.44}$$

By comparing Eqs. (3.43) and (3.44), we can see that Γ_s lies on a circle with center at

$$C_s = \frac{g_s S_{11}^*}{1-(1-g_s)|S_{11}|^2} \tag{3.45}$$

The radius of the circle is given by

$$\begin{aligned} r_s^2 &= |C_s|^2 + \frac{1-|S_{11}|^2-g_s}{1-(1-g_s)|S_{11}|^2} \\ &= \frac{(1-|S_{11}|^2-g_s)[1-(1-g_s)|S_{11}^2|] + g_s^2|S_{11}|^2}{[1-(1-g_s)|S_{11}|^2]^2} \\ &= \frac{(1-g_s)(|S_{11}|^4 - 2|S_{11}|^2+1)}{[1-(1-g_s)|S_{11}|^2]^2} \\ &= \frac{(1-g_s)(1-|S_{11}|^2)^2}{[1-(1-g_s)|S_{11}|^2]^2} \end{aligned} \tag{3.46}$$

so that

$$r_s = \frac{\sqrt{1-g_s}(1-|S_{11}|^2)}{1-(1-g_s)|S_{11}|^2} \tag{3.47}$$

The center C_L and radius r_L of the Γ_L circle for a constant value of g_L can be obtained from Eqs. (3.45) and (3.47) by replacing g_s with g_L and S_{11} with S_{22}.

These results lead to a design procedure for a desired value of the gain:

1. From $G_{TU} = G_s|S_{21}|^2 G_L$, determine desired values for G_s and G_L. One approach is to conjugate match the input port so that $G_s = G_{s,\max}$, and then choose G_L to obtain the desired gain.
2. Compute the normalized gains g_s and g_L.
3. Compute the load gain circle center and radius, C_L and r_L. If the source is not conjugate matched, compute the source gain circle center and radius C_s and r_s also.
4. Choose convenient values of Γ_s and Γ_L on these circles. Any value on the circle will meet the gain target, so we need some way to choose a particular value. One possibility is to choose the reflection coefficient with smallest magnitude ($|\Gamma|$ closest to zero). Later, we will consider other performance metrics such as noise figure that will dictate the choices of Γ_s and Γ_L.

5. Find the source network impedance and load network impedance using

$$Z_s = Z_o \frac{1+\Gamma_s}{1-\Gamma_s} \quad (3.48)$$

$$Z_L = Z_o \frac{1+\Gamma_L}{1-\Gamma_L} \quad (3.49)$$

If $g_s = 1$, then $C_s = S_{11}^*$ and $r_s = 0$, so that $\Gamma_s = S_{11}^*$, which is what we expect, since this is a conjugate match.

If we conjugate match both the input and output, then we can have both $G_s > 1$ and $G_L > 1$. How is this possible with passive matching networks?

3.3 Stability

After gain, the next key amplifier concept is stability. If an amplifier is unstable, noise feedback will lead to oscillation. Stability can be determined from the reflection coefficients Γ_{in} and Γ_{out} looking into the input and output of the transistor. If the magnitude of one or both of these reflection coefficients is greater than unity, then the amplifier is unstable.

Since Γ_{in} and Γ_{out} depend on the reflection coefficients Γ_s and Γ_L looking from the device into the source and load, the matching networks determine the stability of the amplifier. There are two possible situations:

1. Unconditional stability: $|\Gamma_{\text{in}}| < 1$ and $|\Gamma_{\text{out}}| < 1$ for all passive source and load impedances ($|\Gamma_s| < 1$, $|\Gamma_L| < 1$).
2. Conditional stability: $|\Gamma_{\text{in}}| < 1$ and $|\Gamma_{\text{out}}| < 1$ only for a certain range of source and load impedances. This is also called potentially unstable. For this case, we design the source and load matching networks to be such that the amplifier is in the stable region.

In order to determine the stability of an amplifier, we need to examine the reflection coefficients looking into the two device ports. The stability conditions are

$$|\Gamma_{\text{in}}| = \left| S_{11} + \frac{S_{12}\Gamma_L S_{21}}{1 - S_{22}\Gamma_L} \right| < 1 \tag{3.50}$$

$$|\Gamma_{\text{out}}| = \left| S_{22} + \frac{S_{12}\Gamma_s S_{21}}{1 - S_{11}\Gamma_s} \right| < 1 \tag{3.51}$$

We will use these to find out what values Γ_s and Γ_L may take on in order to have stability.

Unilateral device. For a unilateral device, $S_{12} = 0$, and the stability conditions become

$$|S_{11}| < 1 \tag{3.52}$$

$$|S_{22}| < 1 \tag{3.53}$$

which are a function of the device only, and not the source and load matching networks.

Bilateral device. The bilateral case is more complicated, because stability depends on the source and load matching networks. The boundary of the region of stability is defined by

$$|\Gamma_{\text{in}}| = |\Gamma_{\text{out}}| = 1 \tag{3.54}$$

The first condition becomes

$$|S_{11}(1 - S_{22}\Gamma_L) + S_{12}\Gamma_L S_{21}| = |1 - S_{22}\Gamma_L| \tag{3.55}$$

$$|S_{11} - \Delta\Gamma_L| = |1 - S_{22}\Gamma_L| \tag{3.56}$$

where $\Delta = S_{11}S_{22} - S_{12}S_{21}$ is the determinant of the S-parameter matrix of the device. Squaring both sides of the last expression leads to

$$|S_{11}|^2 + |\Delta|^2|\Gamma_L|^2 - \Delta\Gamma_L S_{11}^* + \Delta^*\Gamma_L^* S_{11} = 1 + |S_{22}|^2|\Gamma_L|^2 - (S_{22}^*\Gamma_L^* + S_{22}\Gamma_L) \tag{3.57}$$

Combining terms containing Γ_L gives

$$|\Gamma_L|^2 - \frac{(S_{22} - \Delta S_{11}^*)\Gamma_L + (S_{22}^* - \Delta^* S_{11})\Gamma_L^*}{|S_{22}|^2 - |\Delta|^2} = \frac{|S_{11}|^2 - 1}{|S_{22}|^2 - |\Delta|^2} \tag{3.58}$$

If we compare this to Eq. (3.44) for a circle in the complex plane, we find that the center and radius of the circle in the Γ_L plane is

$$C_L = \frac{S_{22}^* - \Delta^* S_{11}}{|S_{22}|^2 - |\Delta|^2} \tag{3.59}$$

$$r_L = \left| \frac{S_{12} S_{21}}{|S_{22}|^2 - |\Delta|^2} \right| \tag{3.60}$$

For the condition on Γ_{out}, we get similar expressions for the center C_s and radius r_s of the stability circle in the Γ_s plane, with S_{11} and S_{22} interchanged. Why does the input stability condition lead to a circle for Γ_L, which is on the output side, and the output stability condition lead to a condition on Γ_s, the reflection coefficient looking into the source network?

If we have a matched load, then $Z_L = Z_0$ and $\Gamma_L = 0$, so that

$$|\Gamma_{\text{in}}| = |S_{11}| \tag{3.61}$$

If $|S_{11}| < 1$, the center of the Smith chart represents a stable value of Γ_L. Otherwise, the center of the Smith chart is in the unstable region. This can be used to determine whether the inside or the outside of a stability circle represents the stable region for a device.

In practice, it is good to be well inside the stable region, and to be sure that Γ_s and Γ_L are inside the stable region over a range of frequencies near the design frequency.

Unconditional stability. If a device is unconditionally stable, then the entire Smith chart ($|\Gamma| < 1$) is inside the stability circles. It can be shown that a device is unconditionally stable if

$$\frac{1 - |S_{11}|^2}{|S_{22} - S_{11}^* \Delta| + |S_{12} S_{21}|} > 1 \tag{3.62}$$

and the greater this quantity is, the more stable the device.

3.4 Bilateral Design

If $S_{12} \neq 0$, the constant gain circle design approach used above needs some changes. For a bilateral device, the transducer gain cannot be separated into independent factors for the source and load ports, so to achieve a given value of the gain we would have to adjust both Γ_s and Γ_L at the same time. To avoid this difficulty, we will work with either the power gain G_P or the available power gain G_A since they are independent of the source or load, respectively.

The power gain is

$$G_P = \frac{P_L}{P_{\text{in}}} = \frac{1}{1 - |\Gamma_{\text{in}}|^2} |S_{21}|^2 \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2} \tag{3.63}$$

The goal here is a way to find Γ_L given a specified power gain. First, we need to write Γ_{in} in terms of Γ_L, using

$$|\Gamma_{\text{in}}| = \left| S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} \right| = \left| \frac{S_{11} - S_{11}S_{22}\Gamma_L + S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} \right| = \left| \frac{S_{11} - \Delta\Gamma_L}{1 - S_{22}\Gamma_L} \right| \tag{3.64}$$

With this result, the power gain can be expressed as

$$G_P = \frac{1}{1 - \left| \frac{S_{11} - \Delta\Gamma_L}{1 - S_{22}\Gamma_L} \right|^2} |S_{21}|^2 \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2} \tag{3.65}$$

which is a function of Γ_L and the device parameters only. We will use this expression to design the value of Γ_L to achieve a specified power gain, and then use a conjugate match for the source network.

The power gain normalized by the intrinsic transistor gain $|S_{21}|^2$ is

$$\begin{aligned} g_P &= \frac{G_P}{|S_{21}|^2} \\ &= \frac{1 - |\Gamma_L|^2}{1 - S_{22}\Gamma_L - S_{22}^*\Gamma_L^* + |S_{22}\Gamma_L|^2 - |S_{11}|^2 + S_{11}^*\Delta\Gamma_L + S_{11}\Delta^*\Gamma_L^* - |\Delta\Gamma_L|^2} && (3.66) \\ &= \frac{1 - |\Gamma_L|^2}{1 - |S_{11}|^2 + |\Gamma_L|^2(|S_{22}|^2 - |\Delta|^2) - \Gamma_L(S_{22} - \Delta S_{11}^*) - \Gamma_L^*(S_{22}^* - \Delta^* S_{11})} && (3.67) \end{aligned}$$

Cross multiplying and rearranging leads to

$$|\Gamma_L|^2 - \Gamma_L \frac{g_P(S_{22} - \Delta S_{11}^*)}{D} - \Gamma_L^* \frac{g_P(S_{22}^* - \Delta^* S_{11})}{D} + \frac{g_P(1 - |S_{11}|^2)}{D} = \frac{1}{D} \tag{3.68}$$

where $D = 1 + g_P(|S_{22}|^2 - |\Delta|^2)$. We recognize this as a circle in the Γ_L plane. The center and radius are

$$\begin{aligned} C_P &= \frac{g_P(S_{22}^* - \Delta^* S_{11})}{1 + g_P(|S_{22}|^2 - |\Delta|^2)} && (3.69) \\ r_P &= \frac{[1 - 2k|S_{12}S_{21}|g_P + |S_{12}S_{21}|^2 g_P^2]^{1/2}}{|1 + g_P(|S_{22}|^2 - |\Delta|^2)|} && (3.70) \\ k &= \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}S_{21}|} && (3.71) \end{aligned}$$

This is similar to the unilateral design procedure we developed earlier, except that we do not know the range of possible values for g_P.

In order to understand the range of attainable gains in the bilateral case, we need to find out the maximum value of g_P. Let the quantity inside the square brackets in the expression for r_P be denoted by $f(g_P)$, so that

$$f(g_P) = 1 - 2k|S_{12}S_{21}|g_P + |S_{12}S_{21}|^2 g_P^2 \tag{3.72}$$

Since we must have $r_P > 0$, then $f(g_P) > 0$ as well. We can also see that $f(0) = 1$. Using the quadratic formula, the zeros of $f(g_P)$ are

$$g_{1,2} = \frac{1}{|S_{12}S_{21}|}[k \mp \sqrt{k^2 - 1}] \tag{3.73}$$

where g_1 corresponds to the upper sign and g_2 to the lower. Using the zeros, we can factor the polynomial into the product form

$$f(g_P) = |S_{12}S_{21}|^2 \left[g_P - \frac{1}{|S_{12}S_{21}|}(k - \sqrt{k^2 - 1})\right] \left[g_P - \frac{1}{|S_{12}S_{21}|}(k + \sqrt{k^2 - 1})\right] \tag{3.74}$$

$$= |S_{12}S_{21}|^2 (g_P - g_1)(g_P - g_2) \tag{3.75}$$

If we assume that $k > 1$ (which is true for a transistor that is unconditionally stable), then both zeros are positive.

Putting all of this together, we can see that $f(g_P)$ is a parabola that is one at $g_P = 0$, goes negative at g_1, and becomes positive again at g_2. Since $f(g_P)$ must be positive, we can see that the interval for physically meaningful values of the normalized gain is $0 \le g_P \le g_1$. We therefore have

$$g_{P,\max} = \frac{1}{|S_{12}S_{21}|}[k - \sqrt{k^2 - 1}] \tag{3.76}$$

$$G_{P,\max} = \left|\frac{S_{21}}{S_{12}}\right| [k - \sqrt{k^2 - 1}] \tag{3.77}$$

Also, since $r_P = 0$ at $g_P = g_{P,\max}$, C_P becomes

$$\Gamma_{ML} = \frac{g_{P,\max}(S_{22}^* - \Delta^* S_{11})}{1 + g_{P,\max}(|S_{22}|^2 - |\Delta|^2)} \tag{3.78}$$

which is the value of Γ_L that maximizes G_P. These results provide a design approach for a bilateral transistor amplifier.

Design Procedure

1. For the desired power gain G_P, compute the normalized gain g_p and plot the resulting gain circle on the Γ_L plane.

2. *Load reflection coefficient:* Choose a value of Γ_L on the gain circle.

3. *Source reflection coefficient:* Compute Γ_{in} using the selected value for Γ_L. Conjugate match the source, so that $\Gamma_S = \Gamma_{\text{in}}^*$. For this matching condition, $P_{\text{in}} = P_{\text{avs}}$ and therefore $G_T = G_P$.

Our choice of Γ_L produces a value for Γ_{in} which in turn determines Γ_S. This value of Γ_S results in a value of Γ_{out} which determines the output VSWR. If we don't like the output VSWR that we obtain for some reason, we can always choose a different value of Γ_L.

We won't take the time to prove this, but it can be shown that if we pick $\Gamma_L = \Gamma_{ML}$, then $\Gamma_{\text{out}}^* = \Gamma_L$. In other words, maximizing the gain is equivalent to conjugate matching the input and output.

3.5 Noise in Communications Systems

There are several types of noise that are included in communication systems.

1. Thermal Noise (Johnson or Nyquist noise): Created by thermal vibration of bound charges.
2. Shot Noise: Random fluctuations of charge carriers in a solid-state device.
3. Flicker Noise ($1/f$ noise): Occurs in solid-state components. The noise power varies as $1/f$.
4. Plasma Noise: Random motion of charges in an ionized gas.

Thermal noise tends to be dominant in most systems, so we will concentrate on this.

Consider a resistor with resistance R at a temperature T (in Kelvin). The kinetic energy of the electrons is proportional to T. The random motion of the electrons create voltage fluctuations at the resistor terminals. The voltage has zero average, but the RMS value is given by Planck's blackbody radiation equation

$$\overline{v}_n = \sqrt{\frac{4hfBR}{e^{hf/k_BT} - 1}} \tag{3.79}$$

where

B Bandwidth in Hertz
h Planck's constant = 6.546×10^{-34} J·sec
k_B Boltzmann's constant = 1.380×10^{-23} J/K
f frequency (Hz)

If the frequency is large, say $f = 100$ GHz, and the temperature is low, so that $T = 100$K, then

$$hf = 6.5 \times 10^{-23} \ll k_BT = 1.38 \times 10^{-21} \tag{3.80}$$

This means that the exponent hf/k_BT is very small. The inequality gets even larger for microwave frequencies at room temperature (T = 273 K). Because of this, at microwave frequencies the exponential can be approximated by the first two terms of the Taylor series,

$$e^{hf/k_BT} \approx 1 + \frac{hf}{k_BT} \tag{3.81}$$

This simplifies the RMS voltage to

$$\overline{v}_n \approx \sqrt{\frac{4hfBR}{1 + hf/k_BT - 1}} = \sqrt{4k_BTBR} \tag{3.82}$$

In this approximation, $\overline{v}_n$ is independent of frequency. For this reason, the thermal noise signal is called "white noise". We generally model the noise voltage as a random variable with a zero mean Gaussian distribution and variance $\overline{v}_n^2$. Given multiple noise sources, the distributions are independent. Mathematically, this means that if you combine multiple noise sources, the variance of the sum is equal to the sum of the variances (we add the noise powers, not the voltages).

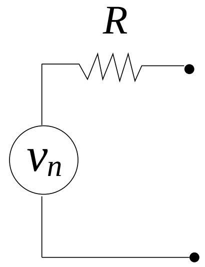

Figure 3.3: Equivalent circuit of a noise source.

We can replace any noisy (warm) resistor with a Thevenin equivalent of a noise source and an ideal, noiseless resistor (Fig. 3.3). If we connect this equivalent circuit to a bandpass filter with bandwidth B Hz and then to a second ideal resistor R (where the resistance of the load is chosedn for maximum power transfer), the noise power delivered to the load is

$$P_n = \left(\frac{\overline{v}_n}{2R}\right)^2 R = \frac{\overline{v}_n^2}{4R} \tag{3.83}$$

Note that we do not have another factor of two in the denominator as we would for phasor voltages since $\overline{v}_n$ is already an RMS quantity. Using our expression for $\overline{v}_n$ leads to

$$P_n = \frac{4k_BTBR}{4R} = k_BTB \tag{3.84}$$

This result is very often used for other noise sources than resistors. The noise source may not even be at a physical temperature equal to T, in which case T in (3.84) becomes an equivalent noise temperature.

When working with microwave signals, it is often convenient to use units of dBm, which means power expressed in decibels relative to 1 milliwatt (dBm is $10\log_{10}$[Power(mW)]). For a resistor at room temperature (approximately $T = 290$ K), $10\log_{10} k_BT = -174$ dBm/Hz. In order to go from this quantity, which measures the amount of noise power in a 1 Hz bandwidth, we multiply by the system bandwidth, or add $10\log_{10} B$ in dB to find the total in-band noise power.

3.5.1 Noise Figure

A key measure of system performance is signal-to-noise ratio (SNR):

$$\text{SNR} = \frac{S}{N} = \frac{\text{Signal Power}}{\text{Noise Power}} \tag{3.85}$$

A high SNR means that it is easy to recognize the signal, and a low SNR means that the signal is obscured by noise.

Amplifiers, lossy transmission lines, mixers, and almost any other component of a microwave system add noise to the signal. An ideal component does not add any noise, so the SNR at the output is the same as the SNR at the input. But for a non-ideal component, the output SNR is always less than the input SNR.

Noise figure is a measure of the degradation in signal-to-noise ratio (SNR) as a signal passes through a system component. The definition of noise figure (F) is the ratio of the total available noise power at the amplifier output to the available noise power at the output due to the input noise only:

$$F = \frac{\text{Output noise power}}{\text{Ideal output noise power = Gain} \times \text{input noise power}} \geq 1 \tag{3.86}$$

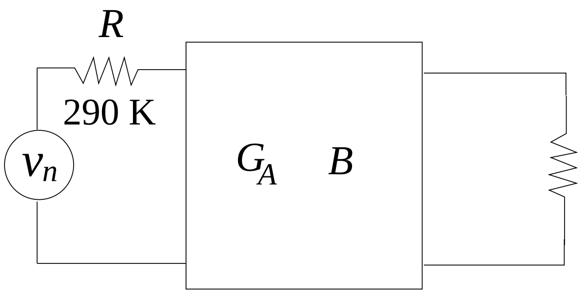

Figure 3.4: Noisy amplifier.

For an ideal component, $F = 1$. The gain used in this expression is the available gain

$$G_A = \frac{P_{\text{avn}}}{P_{\text{avs}}} = \frac{S_o}{S_i} \tag{3.87}$$

It can be seen that noise figure is also equal to the ratio of the input SNR to the output SNR:

$$F = \frac{N_o}{N_i G_A} = \frac{N_o}{N_i S_o/S_i} = \frac{S_i/N_i}{S_o/N_o} = \frac{\text{SNR}_{\text{in}}}{\text{SNR}_{\text{out}}} \tag{3.88}$$

We can also write

$$F = \frac{G_A N_i + P_n}{G_A N_i} = 1 + \frac{P_n}{G_A N_i} \tag{3.89}$$

where P_n is the extra noise power at the output introduced by the component. As a convention, we assume that the input noise corresponds to room temperature, so that $N_i = k_B T_0 B$ with $T_0 = 290\,\text{K}$. Since noise figure is a dimensionless quantity, it is often expressed in dB.

3.5.2 Equivalent Noise Temperature

We can also express the "noisiness" of a component in terms of an equivalent noise temperature using $P = k_B T B$. If we consider an ideal, noiseless component with a warm resistor at the input, then the equivalent temperature T_e is defined to be the temperature of the resistor such that it supplies the same noise as the non-ideal component, so that

$$P_n = G_A k_B T_e B \tag{3.90}$$

Using this in Eq. (3.89) together with $N_i = k_B T_0 B$ leads to

$$F = 1 + \frac{T_e}{T_0} \tag{3.91}$$

Equivalent temperature is often used for very low noise figure devices.

3.5.3 Lossy Components

A lossy system component such as a length of lossy transmission line leads to a degradation in SNR. The basic principle for determining the noise figure of a lossy component is to realize that the noise power at the output of the component must be the same as the noise power at the input (thermal equilibrium), so that

$$G N_i + P_n = N_i \tag{3.92}$$

Solving for the equivalent additional power at the input gives $P_n = N_i(1 - G)$. The noise figure is then

$$F = 1 + \frac{P_n}{GN_i} = 1 + \frac{N_i(1 - G)}{GN_i} = \frac{1}{G} = L \tag{3.93}$$

where L is the power loss of the device. Thus, the noise figure is the same as the loss.

3.5.4 Cascaded Networks

If we have two stages in a system,

$$N_o = G_{A2}N_{o1} + P_{n2} = G_{A2}(G_{A1}N_i + P_{n1}) + P_{n2} \tag{3.94}$$

$$F = \frac{G_{A2}(G_{A1}N_i + P_{n1}) + P_{n2}}{N_i G_{A1} G_{A2}} = 1 + \frac{P_{n1}}{N_i G_{A1}} + \frac{P_{n2}}{N_i G_{A1} G_{A2}} \tag{3.95}$$

In terms of the noise figures of the two stages,

$$F_1 = 1 + \frac{P_{n1}}{N_i G_{A1}} \tag{3.96}$$

$$F_2 = 1 + \frac{P_{n2}}{N_i G_{A2}} \tag{3.97}$$

the noise figure of the system is

$$F = F_1 + \frac{F_2 - 1}{G_{A1}} \tag{3.98}$$

The noise figure of the second state is divided by the gain of the first stage. We can see that the first stage is most critical in determining the noise figure of the system. The idea is that we want to boost the signal as much as possible early in the system while adding as little possible noise so that the signal is larger than noise added by subsequent components in the system. For a receive antenna, for example, we want to have an amplifier with high gain and a noise figure close to unity before a long length of lossy coaxial cable.

3.6 Low Noise Amplifiers

For an amplifier, it can be shown that

$$F = F_{\min} + \frac{R_N}{G_s}|Y_s - Y_{\text{opt}}|^2 \tag{3.99}$$

where

Y_s = $G_s + jB_s$ = source admittance
Y_{opt} = optimum source admittance resulting in minimum noise figure
$F_{\min}$ = minimum noise figure
R_N = equivalent noise resistance of the transistor

Y_{opt}, $F_{\min}$, and R_N are noise parameters for the transistor, and would typically be measured or included in a spec sheet for the transistor.

We want to put Eq. (3.99) in terms of reflection coefficients rather than admittances. Using

$$Y_s = \frac{1}{Z_o}\frac{1-\Gamma_s}{1+\Gamma_s}, \qquad Y_{\text{opt}} = \frac{1}{Z_o}\frac{1-\Gamma_{\text{opt}}}{1+\Gamma_{\text{opt}}} \tag{3.100}$$

the magnitude squared term in Eq. (3.99) becomes

$$\begin{aligned}
|Y_s - Y_{\text{opt}}|^2 &= \frac{1}{Z_o^2}\left|\frac{1-\Gamma_s}{1+\Gamma_s} - \frac{1-\Gamma_{\text{opt}}}{1+\Gamma_{\text{opt}}}\right|^2 \\
&= \frac{1}{Z_o^2}\left|\frac{1-\Gamma_s+\Gamma_{\text{opt}}-\Gamma_s\Gamma_{\text{opt}} - 1 - \Gamma_s + \Gamma_{\text{opt}} + \Gamma_s\Gamma_{\text{opt}}}{(1+\Gamma_s)(1+\Gamma_{\text{opt}})}\right|^2 \\
&= \frac{1}{Z_o^2}\left|\frac{-2\Gamma_s + 2\Gamma_{\text{opt}}}{(1+\Gamma_s)(1+\Gamma_{\text{opt}})}\right|^2 \\
&= \frac{4}{Z_o^2}\left|\frac{\Gamma_s - \Gamma_{\text{opt}}}{(1+\Gamma_s)(1+\Gamma_{\text{opt}})}\right|^2
\end{aligned} \tag{3.101}$$

The source conductance is

$$\begin{aligned}
G_s = \text{Re}\{Y_s\} &= \frac{1}{2}(Y_s + Y_s^*) \\
&= \frac{1}{2Z_o}\left[\frac{1-\Gamma_s}{1+\Gamma_s} + \frac{1-\Gamma_s^*}{1+\Gamma_s^*}\right] \\
&= \frac{1}{2Z_o}\left[\frac{1-\Gamma_s+\Gamma_s^* - |\Gamma_s|^2 + 1 + \Gamma_s - \Gamma_s^* - |\Gamma_s|^2}{|1+\Gamma_s|^2}\right] \\
&= \frac{1}{Z_o}\frac{1-|\Gamma_s|^2}{|1+\Gamma_s|^2}
\end{aligned} \tag{3.102}$$

Using these expressions, the amplifier noise figure becomes

$$\begin{aligned}
F &= F_{\min} + R_N Z_o \frac{1-|\Gamma_s|^2}{|1+\Gamma_s|^2}\frac{4}{Z_o^2}\left|\frac{\Gamma_s - \Gamma_{\text{opt}}}{(1+\Gamma_s)(1+\Gamma_{\text{opt}})}\right|^2 \\
&= F_{\min} + \frac{4R_N}{Z_o}\frac{|\Gamma_s - \Gamma_{\text{opt}}|^2}{(1-|\Gamma_s|^2)|1+\Gamma_{\text{opt}}|^2}
\end{aligned} \tag{3.103}$$

Now, what we would like is to know the values of Γ_s that give a fixed noise figure. To do this, we first define a noise figure parameter N, which consists of all the factors in (3.103) that do not depend on Γ_s:

$$N = \frac{F - F_{\min}}{4R_N/Z_o}|1 + \Gamma_{\text{opt}}|^2 \tag{3.104}$$

We do this to isolate the terms containing Γ_s, and lump the rest into N. Therefore,

$$\begin{aligned}
(\Gamma_s - \Gamma_{\text{opt}})(\Gamma_s^* - \Gamma_{\text{opt}}^*) &= N(1 - \Gamma_s\Gamma_s^*) \\
|\Gamma_s|^2 - \Gamma_s\Gamma_{\text{opt}}^* - \Gamma_s^*\Gamma_{\text{opt}} + |\Gamma_{\text{opt}}|^2 &= N(1 - |\Gamma_s|^2) \\
|\Gamma_s|^2 - \Gamma_s\frac{\Gamma_{\text{opt}}^*}{1+N} - \Gamma_s^*\frac{\Gamma_{\text{opt}}}{1+N} &= \frac{N - |\Gamma_{\text{opt}}|^2}{1+N} \qquad (3.105)
\end{aligned}$$

Once again, we see this is a circle in the complex plane, with center and radius given by

$$\begin{aligned}
C_F &= \frac{\Gamma_{\text{opt}}}{N+1} \qquad (3.106)\\
r_F &= \frac{\sqrt{N(N+1-|\Gamma_{\text{opt}}|^2)}}{N+1} \qquad (3.107)
\end{aligned}$$

Using these expressions, we can draw gain, stability, and noise figure circles on the Γ_s Smith chart and pick a value of Γ_s to achieve multiple specifications.

3.7 Dynamic Range Issues for Amplifiers

There are a few things we need to understand about the power operation of amplifiers.

1. *1 dB Compression Point:* This is defined as the output power at which the gain has dropped 1 dB from its low-power value. Note that the slope of the output versus input power curve is 1 dB/dB. We often denote this point as P_{1dB}.

2. *Dynamic Range:* Range of input that can be detected by the receiver without appreciable distortion. Consider an amplifier with a noise figure F:

$$\begin{aligned}
F &= \frac{N_o}{G_A N_i} = \frac{N_o}{G_A k_B T B} \qquad (3.108)\\
N_o &= F G_A k_B T B \qquad (3.109)
\end{aligned}$$

If the minimum detectable signal for the receiver output (denoted as $S_{o,\text{mds}}$) is X dB above the noise floor, then

$$S_{o,\text{mds}} = -174 \text{ dBm } + 10\log_{10} B + F_{\text{dB}} + X + G_{A,\text{dB}} \tag{3.110}$$

where we have used that $10\log_{10}(10^3 k_B T) = -174$ dBm at $T = 290$ K. The dynamic range is then the difference between the 1 dB compression point P_{1dB} and $S_{o,\text{mds}}$, or

$$DR = P_{\text{1dB}} - S_{o,\text{mds}} = P_{\text{1dB}} + 174 \text{ dBm } - 10\log_{10} B - F_{\text{dB}} - X - G_{A,\text{dB}} \tag{3.111}$$

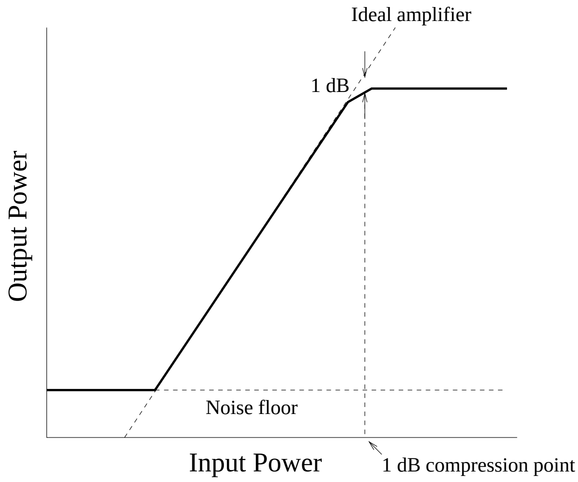

Figure 3.5: Dynamic range of an amplifier.

3. *Third Order Intercept (TOI, TOIP, IP$_3$):* Consider a two-tone test where the input signal is

$$v(t) = A\cos(2\pi f_1 t) + A\cos(2\pi f_2 t) \tag{3.112}$$

where $|f_1 - f_2|$ 5 to 10 MHz. The output frequencies will be of the form

$$f_o = mf_1 + nf_2 \tag{3.113}$$

where m and n are integers. The order of the intermodulation product (IP) is given by $|m| + |n|$.

Note that $2f_1 - f_2$ and $2f_2 - f_1$ will be inside the communication band. The third order intercept point P_{IP} is defines as the output power at which the third order IP power intersects the linear power (assuming no gain compression or saturation occurs). The slope of the third order intermodulation product output power versus input power is 3 dB/dB.

4. *Spurious Free Dynamic Range:* To compute this dynamic range, we continue to use $S_{o,\mathrm{mds}}$ as the lower bound. However, for the upper bound, we take the output power (in the fundamental signal) at which the third order intermodulation product output power reaches $S_{o,\mathrm{mds}}$.