Brigham Young University

Department of Electrical and Computer Engineering

ECEn 464 Sections 001, 002 -- Professor Michael A. Jensen (2-5736)

Exam # 1: November 17-22, 2011

Name: KEY

1. Be sure to be organized in the presentation of your work and to box your answers. A correct answer may not get full credit if I can't easily follow your work.
2. 3"x5" card, calculator, ruler, and compass allowed
3. 3 hour time limit
4. You should have 7 problems, an equation sheet, and 2 Smith charts (12 pages total)

1. ____________

2. ____________

3. ____________

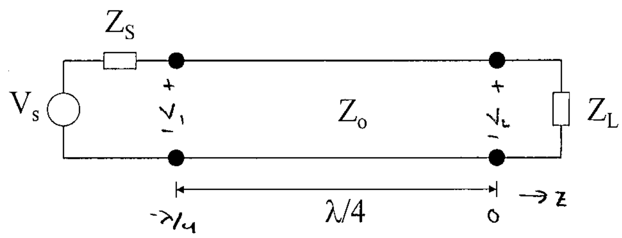
4. ____________

5. ____________

6. ____________

7. ____________

1. For the following circuit, assume that $Z_o = 50\ \Omega$.



a. Determine the voltage across the load (in terms of the source voltage V_s) if $Z_S = Z_o$ and $Z_L = 2Z_o$.

$$Z_{in} = \frac{Z_0^2}{Z_L} = \frac{Z_0^2}{2Z_0} = \frac{Z_0}{2}$$

$$V_1 = \frac{Z_0/2}{Z_0 + Z_0/2} V_s = V^+ \left(e^{-j\beta z} + \Gamma_L e^{j\beta z}\right)\Big|_{z=-\lambda/4}$$

$$\frac{1}{3} V_s = V^+ \left(e^{j\pi/2} + \Gamma_L e^{-j\pi/2}\right) = jV^+(1-\Gamma_L)$$

$$V^+ = \frac{-j}{3(1-\Gamma_L)} V_s$$

$$V_L = V^+\left(e^{-j\beta z} + \Gamma_L e^{j\beta z}\right)\Big|_{z=0} = V^+(1+\Gamma_L) = -\frac{j}{3}\frac{(1+\Gamma_L)}{(1-\Gamma_L)} V_s$$

$$\Gamma_L = \frac{2Z_0 - Z_0}{2Z_0 + Z_0} = \frac{1}{3}$$

$$V_L = -\frac{j}{3}\frac{4/3}{2/3} V_s = -j\frac{2}{3} V_s$$

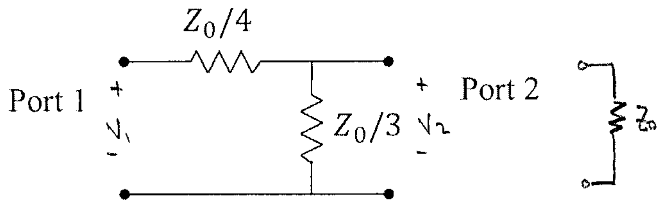
b. Let $Z_L = 2Z_o$. Determine the source impedance Z_S required to maximize the power delivered to the load. Note that the term we use for this is "matched". Does this imply that there are no reflections on the line? Explain.

$Z_{in} = Z_0/2$ (from above)

$Z_S = Z_{in}^* = Z_0/2$

There are reflections, but they work together to achieve maximum power transfer.

2. Compute the S-parameters S_{11}, S_{12}, and S_{21} for the following lumped-element circuit. Use a reference impedance of Z_o. **Will $S_{22} = S_{11}$ for this circuit? Why or why not?**



$$Z_1 = \frac{1}{\frac{3}{Z_0} + \frac{1}{Z_0}} = \frac{Z_0}{4}$$

$$Z_{in} = Z_0/4 + Z_0/4 = Z_0/2$$

$$S_{11} = \frac{Z_0/2 - Z_0}{Z_0/2 + Z_0} = \frac{-1/2}{3/2} = -\frac{1}{3}$$

$$V_2 = \frac{Z_0/4}{Z_0/4 + Z_0/4} V_1 = \frac{1}{2} V_1$$

$$\sqrt{Z_0}\, b_2 = \frac{1}{2}\sqrt{Z_0}\, a_1 (1 + S_{11}) = \frac{1}{2}\sqrt{Z_0}\, a_1 \frac{2}{3}$$

$$S_{21} = \frac{b_2}{a_1} = \frac{1}{3}$$

$$\boxed{\begin{array}{l} S_{11} = -\frac{1}{3} \\ S_{21} = S_{12} = \frac{1}{3} \end{array}}$$

$S_{22} \neq S_{11}$: The circuit is not symmetric.

3. Look at the first Smith Chart attached to this exam, labeled "Problem 3". An impedance point is plotted, labeled "A". Draw on this Smith Chart how the impedance will move for each of the following elements. Do this by drawing the curve between the starting and ending point and placing an arrow at the endpoint of the curve showing the direction that you moved. Label each curve with the letter corresponding to the component ("a", "b", …). It is possible to have multiple letter labels for the same curve.

   a. Series capacitor
   b. Shunt inductor
   c. Open-circuit shunt stub that is shorter than a quarter wavelength – Shut capacitor
   d. Short-circuit shunt stub that is greater than a quarter wavelength but shorter than a half wavelength – Shunt capacitor
   e. Piece of transmission line of characteristic impedance Z_o
   f. Shunt resistor

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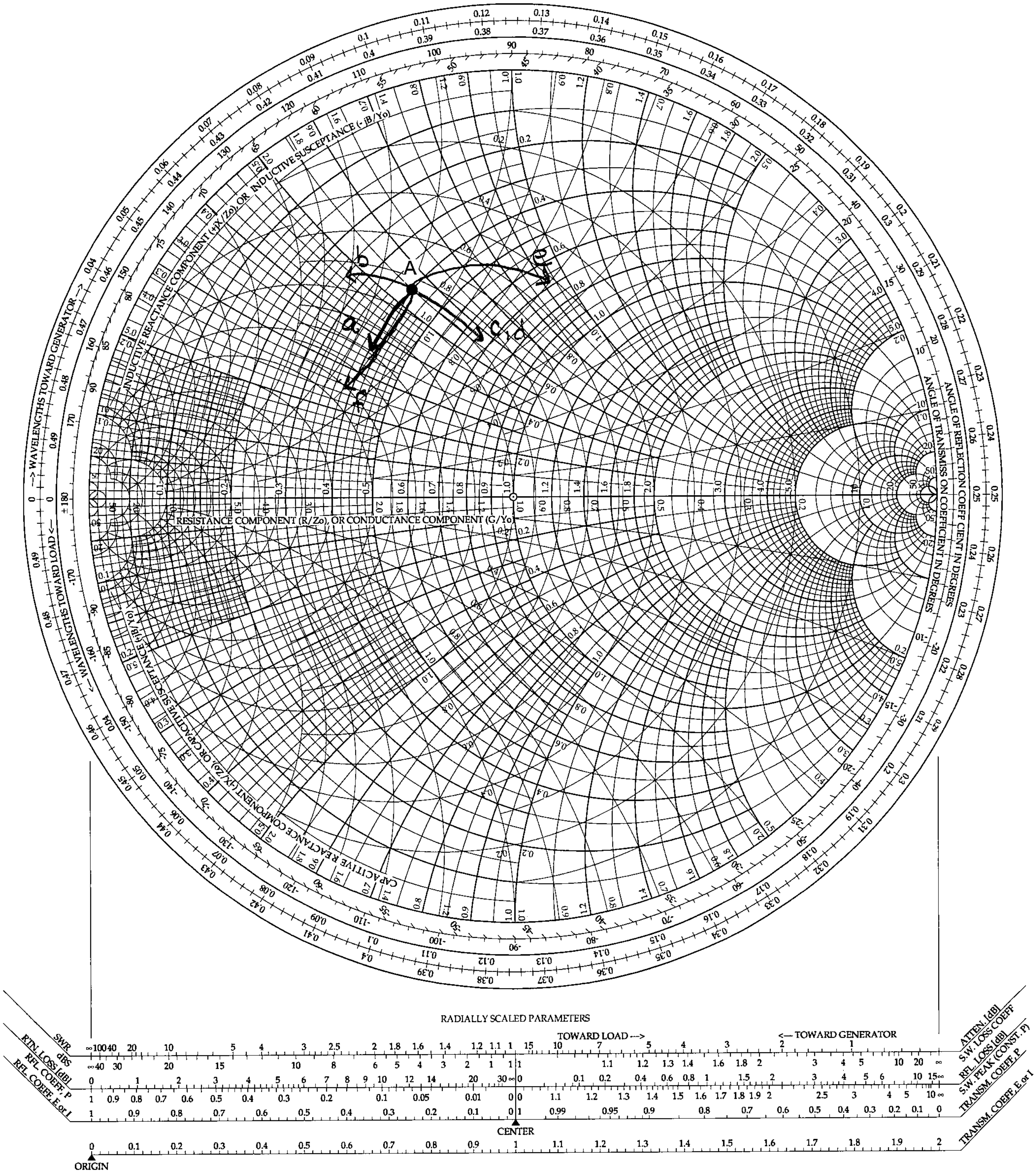
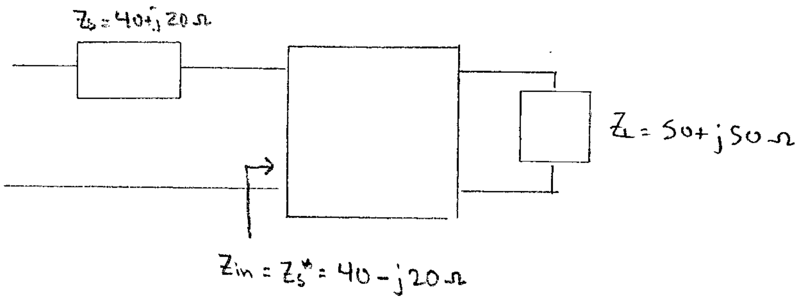
		DATE
SMITH CHART FORM ZY-01-N	COLOR BY J. COLVIN, UNIVERSITY OF FLORIDA, 1997	

Problem 3

NORMALIZED IMPEDANCE AND ADMITTANCE COORDINATES



4. A source with an impedance $Z_S = 40 + j20\ \Omega$ is to be connected to a load with impedance $Z_L = 50 + j50\ \Omega$. Design a 2-element reactive (i.e. consists of inductors and/or capacitors) network to match the load to the source for maximum power transfer. Specify the element values at 500 MHz. A blank Smith Chart is at the end of your exam.

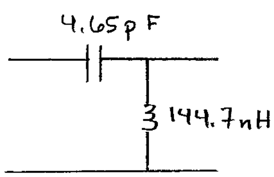


Used $Z_0 = 50$ for Smith Chart

Take Z_L to $Z_{in} = Z_s^*$ on Smith Chart. There are 3 choices.

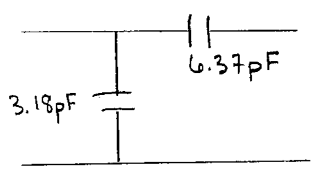
(A) Shunt L: $b = -0.11 = -Z_0/\omega L \Rightarrow L = Z_0/.11\omega = 144.7\text{ nH}$

Series C: $x = -.4 - .97 = -1.37 = -\frac{1}{\omega C Z_0} \Rightarrow C = \frac{1}{1.37\,\omega Z_0} = 4.65\text{ pF}$



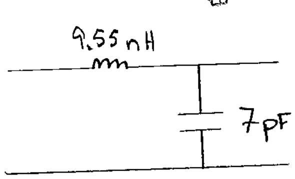
(B) Series C: $x = -1 = -\frac{1}{\omega C Z_0} \Rightarrow C = \frac{1}{\omega Z_0} = 6.37\text{ pF}$

Shunt C: $b = 0.5 = \omega C Z_0 \Rightarrow C = \frac{0.5}{\omega Z_0} = 3.18\text{ pF}$



(C) Shunt C: $b = 0.5 + 0.6 = 1.1 = \omega C Z_0 \Rightarrow C = \frac{1.1}{\omega Z_0} = 7\text{ pF}$

Series L: $x = 0.6 = \frac{\omega L}{Z_0} \Rightarrow L = \frac{0.6 Z_0}{\omega} = 9.55\text{ nH}$



NAME	TITLE	DWG. NO.
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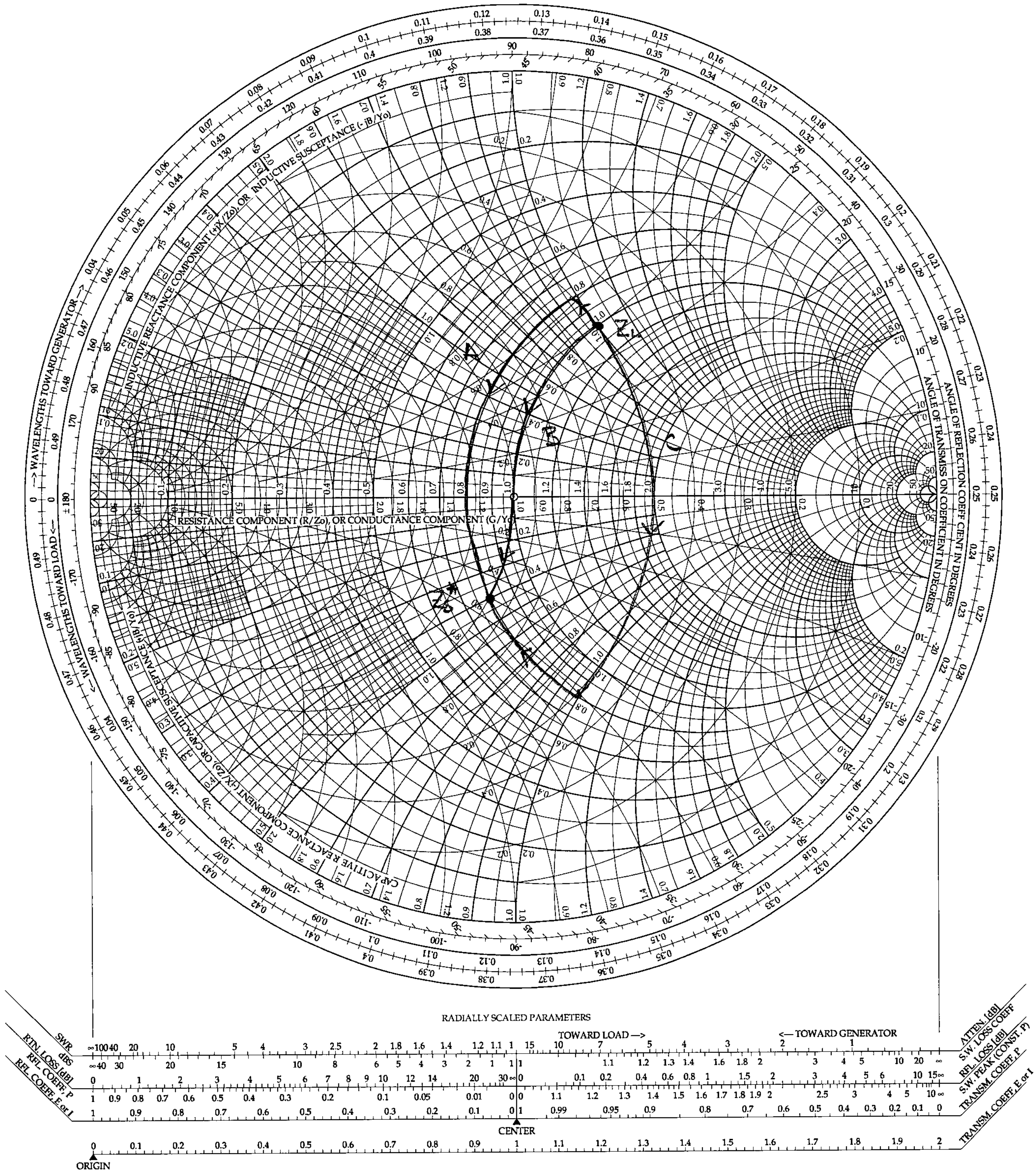
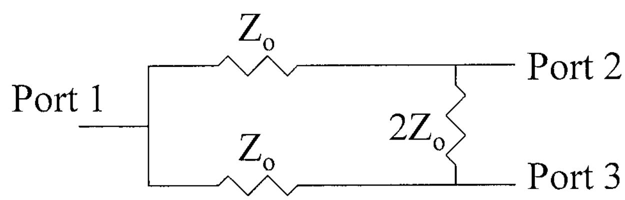
SMITH CHART FORM ZY-01-N	COLOR BY J. COLVIN, UNIVERSITY OF FLORIDA, 1997	

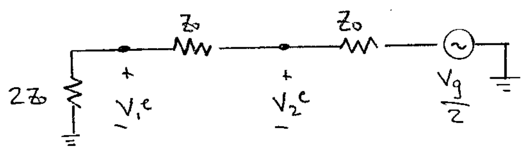
NORMALIZED IMPEDANCE AND ADMITTANCE COORDINATES



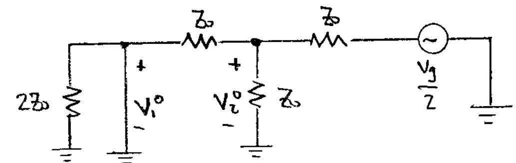
5. Assume a system impedance of Z_o for the following circuit.



a. Draw the even and odd mode equivalent circuits appropriate for computing the S-parameters when a source is placed on port 2.



Even Mode



Odd Mode

b. Use the equivalent circuits to compute the even mode voltages and odd mode voltages at all three ports for excitation on port 2.

$$V_1^e = \frac{2Z_o}{4Z_o}\frac{V_g}{2} = \frac{V_g}{4}$$

$$V_2^e = \frac{3Z_o}{4Z_o}\frac{V_g}{2} = \frac{3V_g}{8}$$

$$V_3^e = V_2^e = \frac{3V_g}{8}$$

$$V_1^o = 0$$

$$V_2^o = \frac{Z_o/2}{Z_o + Z_o/2}\frac{V_g}{2} = \frac{1}{3}\frac{V_g}{2} = \frac{V_g}{6}$$

$$V_3^o = -V_2^o = -\frac{V_g}{6}$$

6. Answer each of the following short questions:

   a. Provide a mathematical (**just involving the powers, not the S-parameters**) definition of the Available Power Gain G_A. Does this gain take into account mismatches at the load, source, or both?

   $$G_A = \frac{P_{avn}}{P_{avs}} = \frac{\text{Power available from network}}{\text{Power available from source}}$$

   Takes into account mismatch at source

   b. Define the S-parameter S_{ij} in terms of physical voltage quantities at ports i and j. Be sure to include any restrictions on the network topology relevant to your definition.

   $$S_{ij} = \left.\frac{V_i^-}{V_j^+}\right|_{V_k^+=0,\, k\neq j}$$

   This means all other ports terminated in system impedance

   c. True or False: You can have a 4-port device that is reciprocal, lossless, and matched at all ports.

   True

   d. Mathematically write the conditions satisfied by the S-parameter matrix of a lossless device.

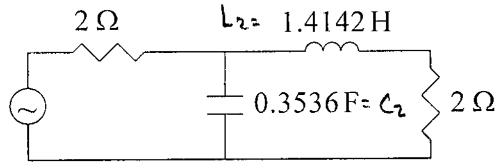
   $$S^T S^* = I$$

   $$\sum_{k=1}^{N} S_{ki} S_{kj}^* = \begin{cases} 1 & i=j \\ 0 & i\neq j \end{cases}$$

   e. Mathematically write the conditions satisfied by the S-parameter matrix of a reciprocal device.

   $$S = S^T$$

7. The following low-pass prototype filter has a cutoff frequency of $\omega_c = 2$ rad/sec. **CAUTION:** The low-pass prototype considered in class has 1Ω source and load resistances and a cutoff frequency of $\omega_c = 1$ rad/sec, so you will need to modify your approach appropriately.

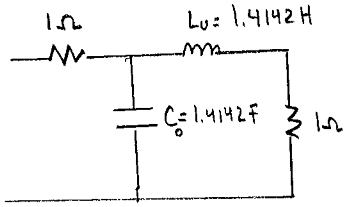


a. Draw the low-pass filter derived from this prototype with a cutoff frequency of $\omega_c = 2 \times 10^9$ rad/sec and source and load impedances of 100 Ω. Specify the component values.

While you can directly scale this, it is easier to first create the low-pass prototype.

$$L_2 = \frac{R_o L_o}{\omega_c} = \frac{2}{2} L_o = L_o$$

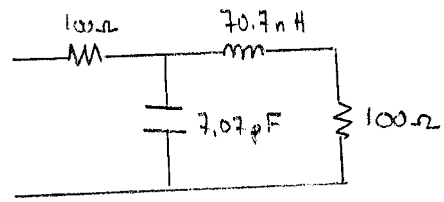
$$C_2 = \frac{C_o}{R_o \omega_c} = \frac{C_o}{4}$$



Now:

$$L = \frac{R_o L_o}{\omega_c} = \frac{100 L_o}{2\times 10^9} = 70.7\ nH$$

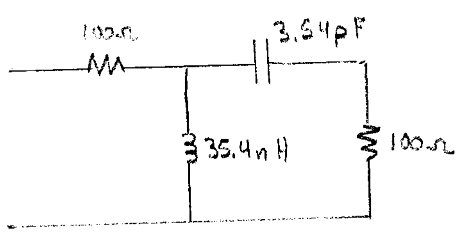
$$C = \frac{C_o}{R_o \omega_c} = \frac{C_o}{200\times 10^9} = 7.07\ pF$$



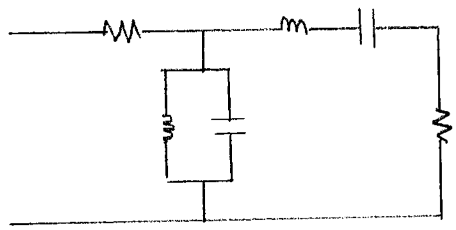
b. Draw the high-pass filter derived from this prototype with a cutoff frequency of $\omega_c = 2 \times 10^9$ rad/sec and source and load impedances of 100 Ω. Specify the component values.

$$L = \frac{R_o}{\omega_c C_o} = \frac{100}{2\times 10^9 C_o} = 35.4\ nH$$

$$C = \frac{1}{R_o \omega_c L_o} = \frac{1}{200\times 10^9 L_o} = 3.54\ pF$$



c. Draw the bandpass filter derived from this prototype. You don't need to specify the component values.



Transmission Lines:

$$\begin{aligned} V(z) &= V^+ e^{-j\beta z} + V^- e^{j\beta z} \\ I(z) &= \frac{V^+}{Z_0} e^{-j\beta z} - \frac{V^-}{Z_0} e^{j\beta z} \end{aligned}$$

$$Z_{\text{in}} = Z_0 \frac{Z_L + jZ_0 \tan \beta\ell}{Z_0 + jZ_L \tan \beta\ell}$$

$$\beta = \omega\sqrt{LC} = \frac{2\pi}{\lambda}$$

S-Parameters:

$$\begin{aligned} V_n^{+'} &= V_n^+ e^{j\theta_n} \\ V_n^{-'} &= V_n^- e^{-j\theta_n} \end{aligned}$$

$$a_n = \frac{V_n^+}{\sqrt{Z_{on}}} \qquad b_n = \frac{V_n^-}{\sqrt{Z_{on}}}$$

Filters (Lowpass Scaling):

$$L'' = \frac{R_0 L}{\omega_c} \qquad C'' = \frac{C}{R_0 \omega_c}$$